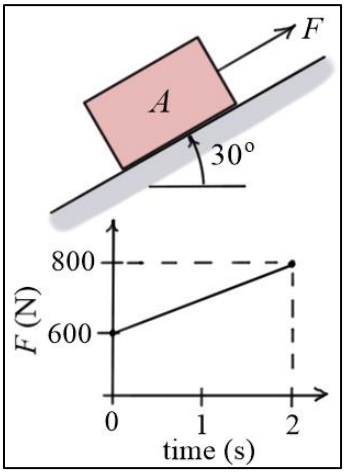
Elementary Dynamics
Exercises #6 – Impulse, Momentum, Impact, and Energy for Particle Motion

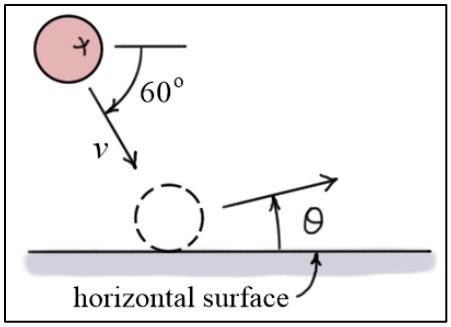
1. The 51 (kg) block *A* is at rest when the force *F* is applied. The block is noted to slide up the inclined plane under the action of the *F*. In the first 2 seconds, *F* varies linearly from 600 (N) to 800 (N). The coefficient of kinetic friction between the block and plane is $\mu_K = 0.5$. Find: a) I_{total} the total impulse applied to *A* by all forces over the 2-second interval, and b) v the velocity of the block after 2 seconds.



   Answers: (positive values are up the incline)

   a) $I_{\text{total}} \approx 466$ (N-s); b) $v \approx 9.15$ (m/s)

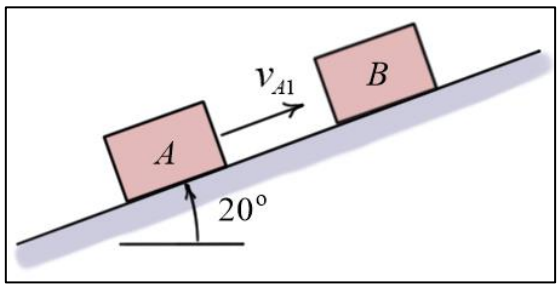
2. A ball of mass $m = 0.1$ (kg) strikes a ***fixed horizontal surface*** with a ***speed*** of $v = 20$ (m/sec) at an ***angle*** of 60 (deg). The ***coefficient of restitution*** for the impact is $e = 0.5$. Neglecting friction and the size of the ball, find: a) θ the ***angle*** at which the ball ***rebounds*** from the surface, and b) $h_{\max}$ the ***maximum height*** the ball attains after leaving the surface. Neglect air resistance.



   Answers:

   a) $\theta \approx 40.9$ (deg); b) $h_{\max} \approx 3.82$ (m)

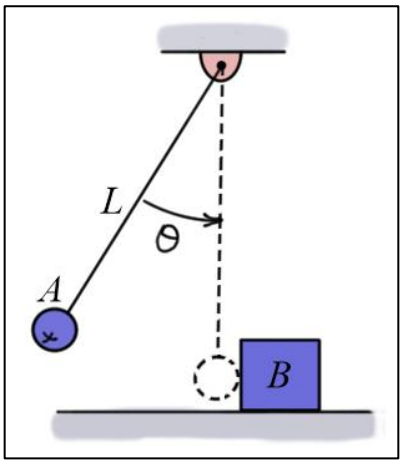
3. At the instant shown, block *B* is at ***rest*** on the inclined plane. ***Just before*** impact, block *A* is traveling up the inclined plane at a speed of $v_{A1} = 30$ (m/s). The masses of the blocks are $m_A = 1$ (kg) and $m_B = 10$ (kg). Find: a) v_{A2} and v_{B2} the velocities of blocks *A* and *B* ***just after*** the impact, and b) Δt the amount of time it takes block *B* to stop. The coefficient of restitution is $e = 0.5$, and the coefficient of kinetic friction is $\mu_K = 0.4$. Neglect ***non-impulsive*** forces during the impact.



   Answers: (positive velocities are up the plane, and negative velocities are down the plane)

   a) $v_{A2} \approx -10.9$ (m/s), $v_{B2} \approx 4.09$ (m/s); b) $\Delta t \approx 0.581$ (s)

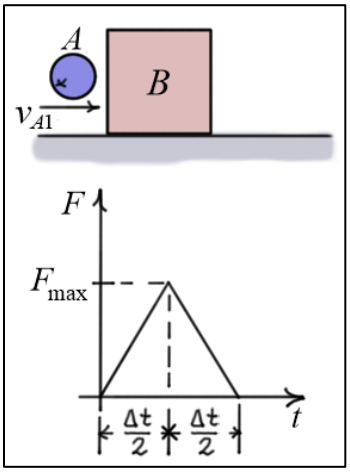
4. The system shown consists of a simple pendulum and a block B on a horizontal surface. Ball A and the block B have masses of $m_A = 2\,(\text{kg})$ and $m_B = 4\,(\text{kg})$, and the pendulum has length $L = 2\,(\text{m})$. As the pendulum swings down, the A strikes B causing it to slide along the surface. The coefficient of restitution between A and B is $e = 0.7$, and the kinetic coefficient of friction between the block and plane is $\mu_k = 0.1$. Given the pendulum is ***released from rest*** at $\theta = 60\,(\text{deg})$, find: a) v_{A1} the velocity of the A ***just before*** it contacts the block, b) v_{A2} and v_{B2} the velocities of A and B ***just after*** the impact, and c) Δt the time it takes for B to come to rest.



Answers: (velocities positive to the right)

a) $v_{A1} \approx 4.43\,(\text{m/s})$; b) $v_{A2} \approx -0.591\,(\text{m/s})$, $v_{B2} \approx 2.51\,(\text{m/s})$; c) $\Delta t \approx 2.56\,(\text{s})$

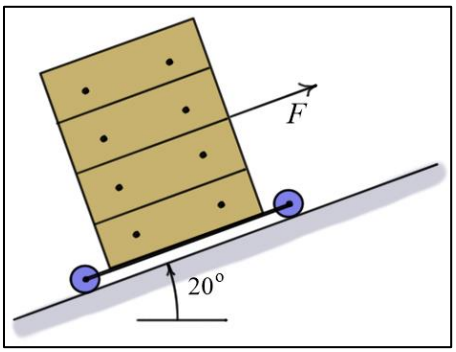
5. Box B is at ***rest*** on the ***horizontal plane*** when it is struck by ***ball*** A causing it to move to the right. The mass of A is $m_A = 1\,(\text{kg})$, the mass of B is $m_B = 5\,(\text{kg})$, the velocity of A just before it strikes B is $v_{A1} = 10\,(\text{m/s})$, the coefficient of restitution is $e = 0.6$, and the coefficient of kinetic friction between B and the floor is $\mu_K = 0.2$. Find: a) v_{B2} the velocity of B just after the impact, and b) t_B the time it takes for B to come to rest after the impact. ***Neglect non-impulsive*** forces during the impact. Then, assuming the impulsive impact force between A and B has the form shown, find c) $F_{\max}$ the ***maximum*** value of the impact force. Assume $\Delta t = 0.006\,(\text{sec})$.



Answers:

a) $v_{B2} \approx 2.67\,(\text{m/s})$; b) $t_B \approx 1.36\,(\text{s})$; c) $F_{\max} \approx 4.44\,(\text{kN})$

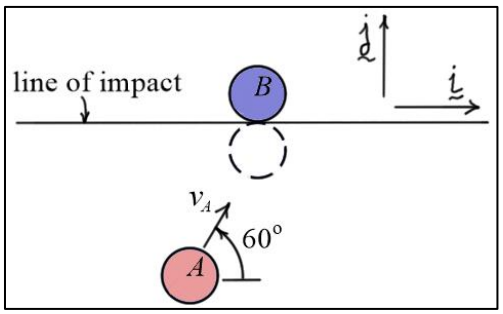
6. The $20\,(\text{lb})$ chest is pulled up the incline by the force $F(t) = (5+t)\,(\text{lb})$, where t is in seconds. Assuming the chest is ***released*** from ***rest***, when F is applied find: a) I_{total} the total impulse applied to the chest over the time interval $0 \le t \le 5$ seconds, and b) v the velocity of the chest when $t = 5\,(\text{s})$. Neglect friction.



Answers: (positive values indicate impulse and velocity up the plane)

a) $I_{\text{total}} \approx 3.30\,(\text{lb-s})$; b) $v \approx 5.31\,(\text{ft/s})$

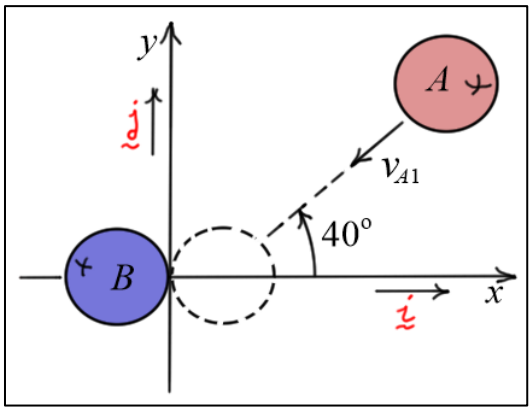
7. Puck A of mass $M = 2$ (kg) slides along a ***horizontal***, ***frictionless*** surface and collides with puck B of mass $m = 1$ (kg) as shown. Just ***before impact***, puck A has velocity $v_A = 5$ (m/s) at the angle shown, and puck B is at ***rest***. The pucks collide so that the line tangent to the impact is along the $\underset{\sim}{i}$ direction as shown. The coefficient of restitution $e = 0.6$. Neglecting friction and the size of the pucks, find the velocities of A and B ***just after impact***.



Answers:

$\underset{\sim}{v}_A \approx 2.5\,\underset{\sim}{i} + 2.02\,\underset{\sim}{j}$ (m/s) and $\underset{\sim}{v}_B \approx 4.62\,\underset{\sim}{j}$ (m/s)

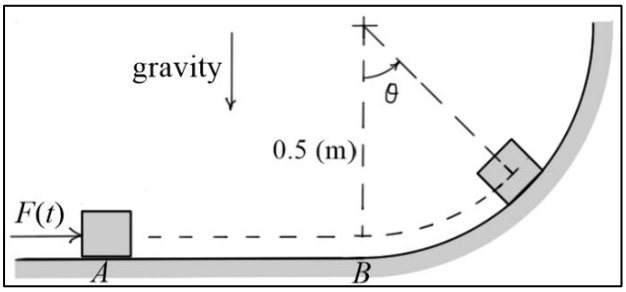
8. The two smooth balls A and B each of mass $m = 0.2$ (kg) are rolling on a horizontal surface. B is at ***rest*** when A strikes it with a velocity $v_{A1} = 1.5$ (m/s) as shown. The coefficient of restitution is $e = 0.85$. ***Neglecting friction*** and the ***size*** of the balls, find: a) $\underset{\sim}{v}_A$ and $\underset{\sim}{v}_B$ the velocities of the balls ***just after*** impact, and b) ϕ the angle that ball A is moving relative to the positive x axis just after the impact.

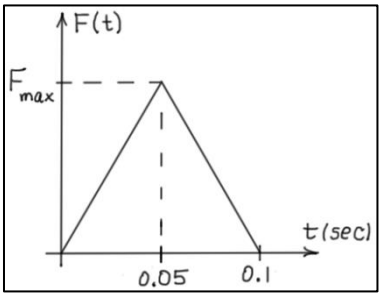


Answers:

a) $\underset{\sim}{v}_A \approx -0.0862\,\underset{\sim}{i} - 0.964\,\underset{\sim}{j}$ (m/s) and $\underset{\sim}{v}_B \approx -1.06\,\underset{\sim}{i}$ (m/s); b) $\phi \approx -95.1$ (deg)

9. A 10 (kg) block is at ***rest*** at A when it is struck with an impulsive force $F(t)$. ***Neglecting friction*** and the ***size*** of the block, find v the velocity of the block at $t = 0.1$ (sec). Assume $F(t)$ varies as shown in the diagram with $F_{\max} = 600$ (N). Does the block stop before reaching the top of the circular track?





Answers:

$v \approx 3$ (m/s)

Yes, the block reaches a maximum height of $0.459\text{ (m)} < 0.5\text{ (m)}$.