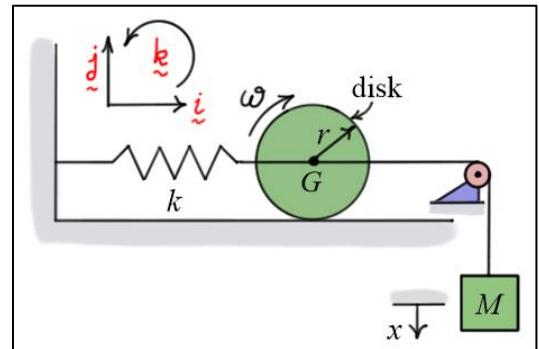


Elementary Dynamics

Exercises #9 – Two-Dimensional Rigid Body Kinetics: Work and Energy

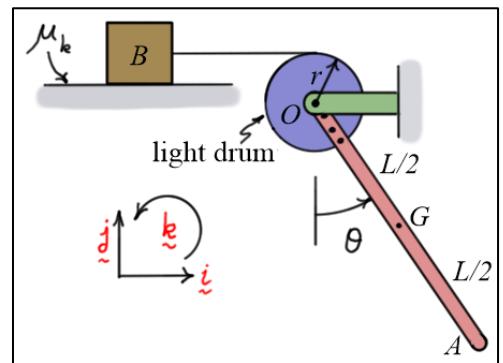
1. The figure shows a system consisting of a 2 (kg) block M and a 5 (kg) disk of radius $r = 0.1$ (m). As the block moves down the disk rolls without slipping on the horizontal plane. The spring has stiffness $k = 56$ (N/m). Given that the system is released from rest with the spring unstretched, find ω the angular velocity of the disk after the block moves down 0.5 meters.



Answer:

$$\omega \approx -7.69 \sqrt{k} \text{ (rad/s)}$$

2. The system shown consists of bar OA , a light drum, block B , and a connecting cable. The bar and drum are riveted together so they rotate as a single body. As the bar and drum rotate about O , block B is pulled along the plane. Bar OA has mass $m = 10$ (kg) and length $L = 2$ (m). Block B has mass $m_B = 15$ (kg). The coefficient of kinetic friction between the block and the plane is $\mu_k = 0.2$. The drum has radius $r = 250$ (mm), and its mass and inertia are negligible.



Given the system is released from rest at $\theta = 30$ (deg), find: a) $U_{1 \rightarrow 2}$ the total work done on the system as it moves from $\theta = 30$ (deg) to $\theta = 0$ (deg), and b) ω_{OA} the angular velocity of OA when it reaches $\theta = 0$.

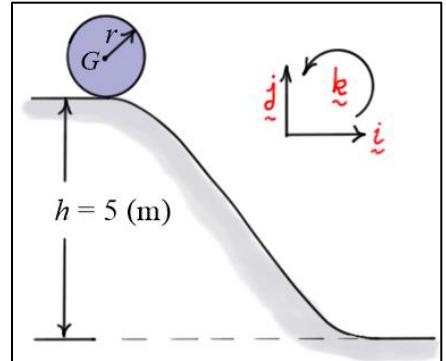
Answers:

a) $U_{1 \rightarrow 2} \approx 9.29$ (N-m); b) $\omega_{OA} \approx -1.14 \sqrt{k}$ (rad/s)

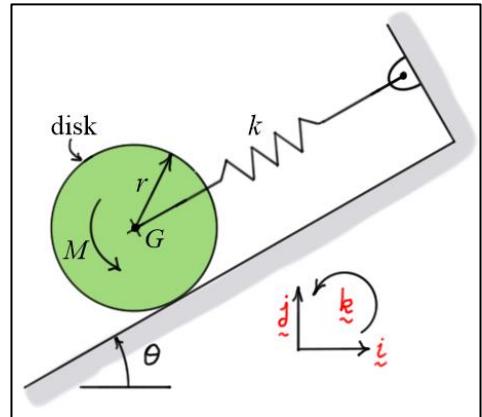
3. The thin disk rolls without slipping down the 5 (m) hill as shown. The disk has mass $m = 7$ (kg) and radius $r = 0.4$ (m). Given it is released from rest at the top of the hill, find ω_D the angular velocity of the disk when it reaches the bottom of the hill.

Answer:

$$\omega_D \approx -20.2 \sqrt{k} \text{ (rad/s)}$$



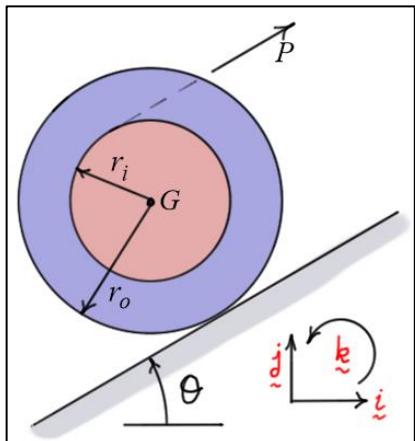
4. The 20-kg disk of radius $r = 0.2$ (m) is initially at rest and the spring is unstretched when the torque of $M = 30$ (N-m) is applied. The plane is inclined at an angle of $\theta = 30$ (deg), and the spring has stiffness $k = 150$ (N/m). Find: a) $U_{1 \rightarrow 2}$ the work done on the system as G moves down the plane 0.8 (m), and b) ω_D the angular velocity of the disk at the instant G arrives at that position. Assume that the disk rolls without slipping.



Answer:

a) $U_{1 \rightarrow 2} \approx 150$ (N-m); b) $\omega_D \approx 15.8\sqrt{k}$ (rad/s)

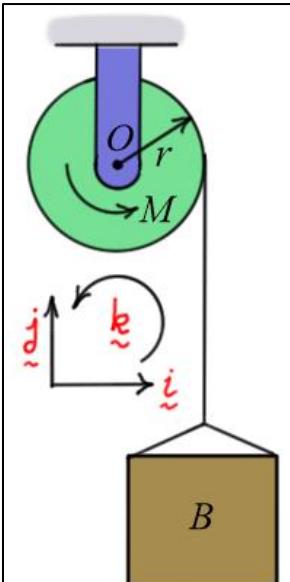
5. The 100 (kg) spool is pulled up the inclined plane ($\theta = 30$ (deg)) by a cable with tension force $P = 500$ (N). The spool starts from rest and rolls without slipping. The inner and outer radii of the spool are $r_i = 0.3$ (m) and $r_o = 0.4$ (m), and the radius of gyration of the spool about its mass center G is $k_G = 0.25$ (m). Find: a) $U_{1 \rightarrow 2}$ the work done on the spool as G moves a distance of 2 (m) up the inclined plane, and b) ω_s the angular velocity of the spool at the instant G arrives at the 2-meter position.



Answers:

a) $U_{1 \rightarrow 2} \approx 769$ (N-m); b) $\omega_s \approx -8.31\sqrt{k}$ (rad/s)

6. A heavy bucket B weighing 1500 (lb) is lifted using a motor and pulley system as shown. The motor applies a constant torque of $M = 2000$ (ft-lb) to the pulley at O . The pulley has radius $r = 1.25$ (ft), radius of gyration about its center $k_O = 0.95$ (ft), and weight $W_p = 115$ (lb). The system starts from rest. Find: a) $U_{1 \rightarrow 2}$ the total work done on the system as the bucket is raised 10 (ft), and b) v_B the velocity of the bucket when it reaches the 10-foot position.



Answers:

a) $U_{1 \rightarrow 2} \approx 1000$ (ft-lb); b) $v_B \approx 6.41\sqrt{k}$ (ft/s)