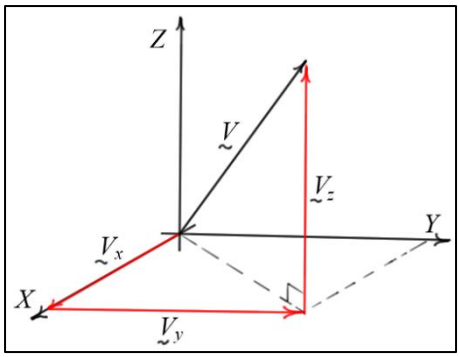
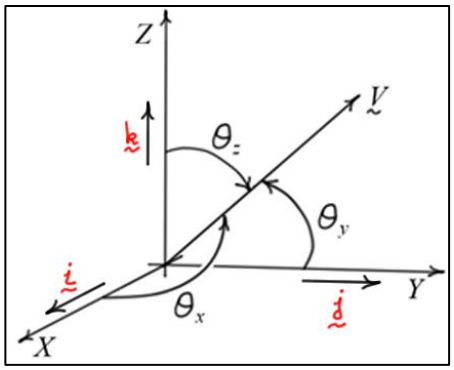
Elementary Statics
Equation Sheet #2: Three Dimensional (3D) Vectors, Dot Product, and Resultant Force

1. 3D vector components:

$$\begin{aligned}\underset{\sim}{V} &= \underset{\sim}{V}_x + \underset{\sim}{V}_y + \underset{\sim}{V}_z \\ &= V_x\,\underset{\sim}{i} + V_y\,\underset{\sim}{j} + V_z\,\underset{\sim}{k} \\ &= \left(V\cos(\theta_x)\right)\underset{\sim}{i} + \left(V\cos(\theta_y)\right)\underset{\sim}{j} + \left(V\cos(\theta_z)\right)\underset{\sim}{k} \\ &= V\left(\cos(\theta_x)\,\underset{\sim}{i} + \cos(\theta_y)\,\underset{\sim}{j} + \cos(\theta_z)\,\underset{\sim}{k}\right) \\ &= V\,\underset{\sim}{u}_V\end{aligned}$$



2. Unit vector along $\underset{\sim}{V}$: $\underset{\sim}{u}_V = \cos(\theta_x)\,\underset{\sim}{i} + \cos(\theta_y)\,\underset{\sim}{j} + \cos(\theta_z)\,\underset{\sim}{k}$



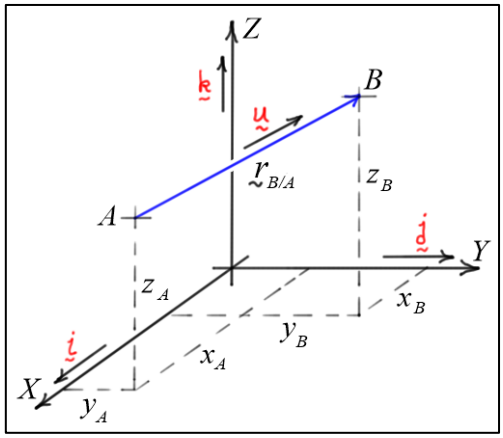
3. Trigonometric identity: $\cos^2(\theta_x) + \cos^2(\theta_y) + \cos^2(\theta_z) = 1$

4. Magnitude: $V = |\underset{\sim}{V}| = \sqrt{V_x^2 + V_y^2 + V_z^2}$

5. Vector addition: $\underset{\sim}{V} = \underset{\sim}{V}_1 + \underset{\sim}{V}_2 = \left(V_{1x} + V_{2x}\right)\underset{\sim}{i} + \left(V_{1y} + V_{2y}\right)\underset{\sim}{j} + \left(V_{1z} + V_{2z}\right)\underset{\sim}{k}$

6. Position vector:

$$\underset{\sim}{r}_{B/A} = \underset{\sim}{r}_B - \underset{\sim}{r}_A = \left(x_B - x_A\right)\underset{\sim}{i} + \left(y_B - y_A\right)\underset{\sim}{j} + \left(z_B - z_A\right)\underset{\sim}{k}$$



7. Unit vector: $\underset{\sim}{u} = \dfrac{\underset{\sim}{r}_{B/A}}{|\underset{\sim}{r}_{B/A}|}$

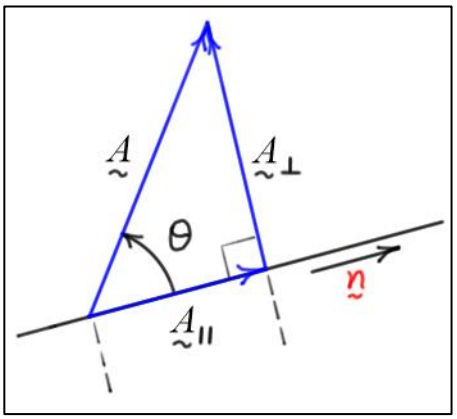
Dot Product

1. Geometric definition: $\underset{\sim}{A}\cdot\underset{\sim}{B} = |\underset{\sim}{A}|\,|\underset{\sim}{B}|\cos(\theta)$

2. Calculation: $\underset{\sim}{A}\cdot\underset{\sim}{B} = \left(a_x\,\underset{\sim}{i} + a_y\,\underset{\sim}{j} + a_z\,\underset{\sim}{k}\right)\cdot\left(b_x\,\underset{\sim}{i} + b_y\,\underset{\sim}{j} + b_z\,\underset{\sim}{k}\right) = a_x b_x + a_y b_y + a_z b_z$

3. Components along and perpendicular to $\underset{\sim}{n}$:

$\underset{\sim}{A}_{\parallel} = \left(\underset{\sim}{A}\cdot\underset{\sim}{n}\right)\underset{\sim}{n}$ and $\underset{\sim}{A}_{\perp} = \underset{\sim}{A} - \underset{\sim}{A}_{\parallel}$



4. Product is ***commutative***: $\underset{\sim}{A}\cdot\underset{\sim}{B} = \underset{\sim}{B}\cdot\underset{\sim}{A}$

5. Product is ***distributive*** over addition: $\underset{\sim}{A}\cdot\left(\underset{\sim}{B} + \underset{\sim}{C}\right) = \left(\underset{\sim}{A}\cdot\underset{\sim}{B}\right) + \left(\underset{\sim}{A}\cdot\underset{\sim}{C}\right)$

6. Multiplication by a ***scalar*** α : $\alpha\left(\underset{\sim}{A}\cdot\underset{\sim}{B}\right) = \left(\alpha\underset{\sim}{A}\right)\cdot\underset{\sim}{B} = \underset{\sim}{A}\cdot\left(\alpha\underset{\sim}{B}\right)$

Resultant Force: $\underset{\sim}{F}_R = \sum_i \underset{\sim}{F}_i$