Elementary Statics
Equation Sheet #3: Particle Equilibrium, Cross Product, and Moments of Forces

Linear Spring Force: $\boxed{F = k\,x}$
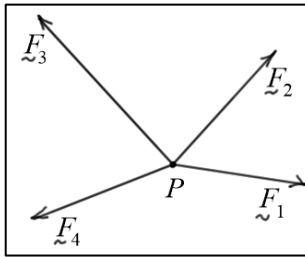
Particle Equilibrium

$$\boxed{\begin{aligned}\sum_i (F_x)_i &= 0 \\ \sum_i (F_y)_i &= 0\end{aligned}} \quad \text{(2D)}$$

$$\boxed{\begin{aligned}\sum_i (F_x)_i &= 0 \\ \sum_i (F_y)_i &= 0 \\ \sum_i (F_z)_i &= 0\end{aligned}} \quad \text{(3D)}$$

Free Body Diagram

Cross Product

1. Geometric Definition: $\boxed{\underset{\sim}{A} \times \underset{\sim}{B} = \left(|\underset{\sim}{A}|\,|\underset{\sim}{B}| \sin(\theta)\right) \underset{\sim}{n}_\perp}$

2. Calculation:

$$\boxed{\underset{\sim}{A} \times \underset{\sim}{B} = \begin{vmatrix} \underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = \left(a_y b_z - a_z b_y\right)\underset{\sim}{i} - \left(a_x b_z - a_z b_x\right)\underset{\sim}{j} + \left(a_x b_y - a_y b_x\right)\underset{\sim}{k}}$$
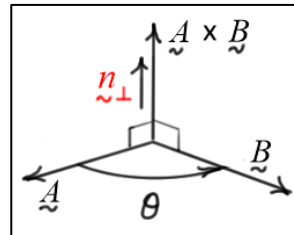
3. Properties of the Cross Product

   o Product is **not** commutative: $\underset{\sim}{A} \times \underset{\sim}{B} = -\left(\underset{\sim}{B} \times \underset{\sim}{A}\right)$

   o Product is distributive over addition: $\underset{\sim}{A} \times \left(\underset{\sim}{B} + \underset{\sim}{C}\right) = \left(\underset{\sim}{A} \times \underset{\sim}{B}\right) + \left(\underset{\sim}{A} \times \underset{\sim}{C}\right)$

   o Multiplication by a scalar α : $\alpha\left(\underset{\sim}{A} \times \underset{\sim}{B}\right) = \left(\alpha\underset{\sim}{A}\right) \times \left(\alpha\underset{\sim}{B}\right)$

Moment of a Force

1. Scalar definition: $\boxed{M_O = F\,d}$
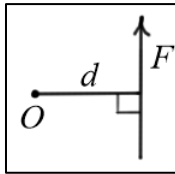
2. Vector definition: $\boxed{\underset{\sim}{M}_O = \underset{\sim}{r} \times \underset{\sim}{F}}$
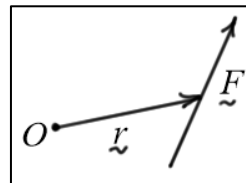
3. Resultant Moment: $\boxed{\left(\underset{\sim}{M}_O\right)_R = \sum_i \left(\underset{\sim}{M}_O\right)_i = \sum_i \left(\underset{\sim}{r}_i \times \underset{\sim}{F}_i\right)}$

Moment of a Force about an Axis a

1. Scalar definition: $\boxed{M_{\underset{\sim}{n}\text{-axis}} = \underset{\sim}{M}_O \cdot \underset{\sim}{n} = \left(\underset{\sim}{r} \times \underset{\sim}{F}\right) \cdot \underset{\sim}{n}}$

2. Vector definition: $\boxed{\underset{\sim}{M}_{\underset{\sim}{n}\text{-axis}} = \left(\underset{\sim}{M}_O \cdot \underset{\sim}{n}\right)\underset{\sim}{n}}$