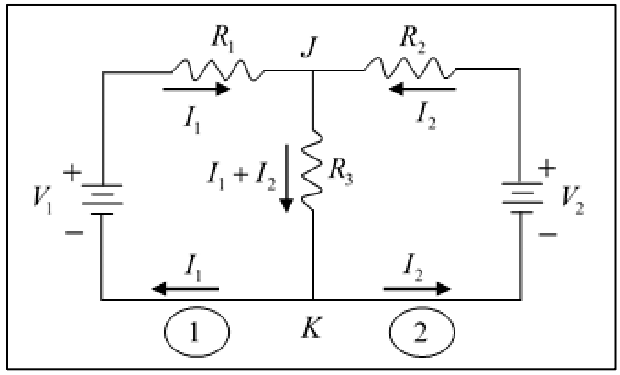
Elementary Engineering Mathematics
Applications of Systems of Linear, Algebraic Equations in Electrical Engineering

Consider the double-loop DC circuit shown in the diagram. Application of ***Kirchhoff's Current Law*** tells us that the current in the middle path (common to both loops) is the ***sum*** of the currents flowing in the outer paths. Using this result, we apply ***Kirchhoff's Voltage Law*** to each loop separately.



$$\boxed{\sum_{\text{loop 1}}\left(\text{voltage rises}\right)=V_1=\sum_{\text{loop 1}}\left(\text{voltage drops}\right)=R_1\,I_1+R_3\left(I_1+I_2\right)}$$

$$\boxed{\sum_{\text{loop 2}}\left(\text{voltage rises}\right)=V_2=\sum_{\text{loop 2}}\left(\text{voltage drops}\right)=R_2\,I_2+R_3\left(I_1+I_2\right)}$$

or

$$\boxed{\begin{aligned}\left(R_1+R_3\right)I_1+\left(R_3\right)I_2&=V_1\\ \left(R_3\right)I_1+\left(R_2+R_3\right)I_2&=V_2\end{aligned}} \qquad (1)$$

Given the resistances and applied voltages, Eqs. (1) can be solved for the currents I_1 and I_2.

<u>Example</u>

<u>Given</u>: In the circuit shown above,

$$R_1=6(\Omega),\ R_2=10(\Omega),\ R_3=5(\Omega),\ V_1=6\ \text{(volts)},\ \text{and}\ V_2=12\ \text{(volts)}$$

<u>Find</u>: currents I_1 and I_2 using a) substitution, and b) Cramer's rule.

<u>Solution:</u>

Using the given values, we have two simultaneous equations: $\boxed{\begin{aligned}11I_1+5I_2&=6\\ 5I_1+15I_2&=12\end{aligned}}$.

a) <u>Substitution:</u>

Solving the first equation for I_2 in terms of I_1, gives $\boxed{I_2=\dfrac{6-11I_1}{5}}$. Substituting this result into the second equation for I_2 gives a single equation that we can solve for I_1.

$$12=5I_1+15\left(\frac{6-11I_1}{5}\right)=5I_1+3\left(6-11I_1\right)=-28I_1+18$$

$$\Rightarrow\quad \boxed{I_1=\frac{18-12}{28}\approx 0.2143\ \text{(amps)}}\quad \text{and}\quad \boxed{I_2=\left(\frac{6-11I_1}{5}\right)\approx 0.7286\ \text{(amps)}}$$

b) <u>Cramer's Rule:</u>

To use Cramer's rule, we first write the equations as a ***matrix equation***.

$$\boxed{\begin{bmatrix} 11 & 5 \\ 5 & 15 \end{bmatrix} \begin{Bmatrix} I_1 \\ I_2 \end{Bmatrix} = \begin{Bmatrix} 6 \\ 12 \end{Bmatrix}}$$

Then, we solve.

$$\boxed{I_1 = \frac{\det \begin{bmatrix} 6 & 5 \\ 12 & 15 \end{bmatrix}}{\det \begin{bmatrix} 11 & 5 \\ 5 & 15 \end{bmatrix}} = \frac{(6 \times 15) - (5 \times 12)}{(11 \times 15) - (5 \times 5)} = \frac{30}{140} \approx 0.2143 \text{ (amps)}}$$

$$\boxed{I_2 = \frac{\det \begin{bmatrix} 11 & 6 \\ 5 & 12 \end{bmatrix}}{\det \begin{bmatrix} 11 & 5 \\ 5 & 15 \end{bmatrix}} = \frac{(11 \times 12) - (6 \times 5)}{(11 \times 15) - (5 \times 5)} = \frac{102}{140} \approx 0.7286 \text{ (amps)}}$$