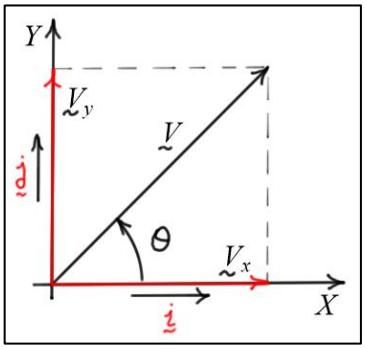
Elementary Engineering Mathematics
Equation Sheet #2 – Two Dimensional Vectors & Complex Numbers

2D Vectors



1. Vector Magnitude and Direction

$$\left|\underset{\sim}{V}\right| = \sqrt{V_x^2 + V_y^2} \qquad \theta = \tan^{-1}\left(V_y / V_x\right)$$

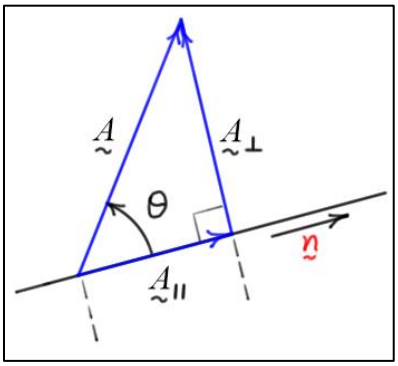
2. Vector Sum

$$\underset{\sim}{V}_1 + \underset{\sim}{V}_2 + \cdots + \underset{\sim}{V}_n = \left(V_{1x} + V_{2x} + \cdots + V_{nx}\right)\underset{\sim}{i} + \left(V_{1y} + V_{2y} + \cdots + V_{ny}\right)\underset{\sim}{j}$$

3. Scalar (Dot) Product of Two Vectors

$$\underset{\sim}{A}\cdot\underset{\sim}{B} = \left|\underset{\sim}{A}\right|\left|\underset{\sim}{B}\right|\cos(\underset{\sim}{A},\underset{\sim}{B}) = \left(a_x\,\underset{\sim}{i} + a_y\,\underset{\sim}{j}\right)\cdot\left(b_x\,\underset{\sim}{i} + b_y\,\underset{\sim}{j}\right) = a_x b_x + a_y b_y$$

4. Vector Components Parallel and Perpendicular to a Unit Vector $\underset{\sim}{n}$

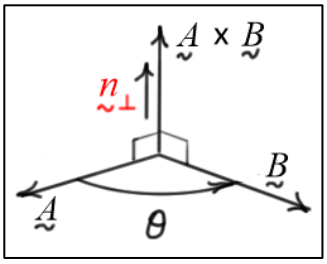


$$\underset{\sim}{A}_{||} = \left(\underset{\sim}{A}\cdot\underset{\sim}{n}\right)\underset{\sim}{n} \quad \text{and} \quad \underset{\sim}{A}_\perp = \underset{\sim}{A} - \underset{\sim}{A}_{||}$$

5. Vector (Cross) Product of Two Vectors

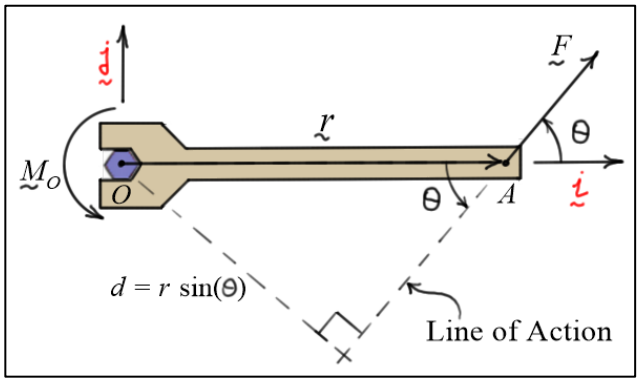
$$\underset{\sim}{A}\times\underset{\sim}{B} = \left(\left|\underset{\sim}{A}\right|\left|\underset{\sim}{B}\right|\sin(\underset{\sim}{A},\underset{\sim}{B})\right)\underset{\sim}{n}_\perp = \left(a_x\,\underset{\sim}{i} + a_y\,\underset{\sim}{j}\right)\times\left(b_x\,\underset{\sim}{i} + b_y\,\underset{\sim}{j}\right) = \left(a_x b_y - a_y b_x\right)\underset{\sim}{k}$$

$$\underset{\sim}{A}\times\underset{\sim}{B} = \begin{vmatrix} \underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k} \\ a_x & a_y & 0 \\ b_x & b_y & 0 \end{vmatrix} = \left(a_x b_y - a_y b_x\right)\underset{\sim}{k}$$

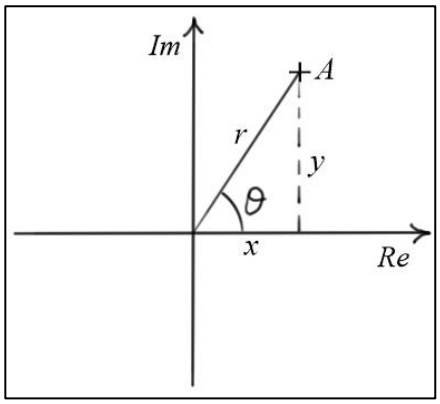


6. Moment of a Force

$$\underset{\sim}{M}_O = \underset{\sim}{r}\times\underset{\sim}{F} \qquad d = \frac{\left|\underset{\sim}{M}_O\right|}{\left|\underset{\sim}{F}\right|}$$



Complex Numbers $\left(j = \sqrt{-1}\right)$



1. Magnitude and Direction

$$A = x + j\,y = r\,e^{j\theta} = r\angle\theta \qquad |A| = r = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1}\left(y/x\right) \qquad e^{j\theta} = \cos(\theta) + j\sin(\theta)$$

2. Sum

$$A_1 + A_2 + \cdots + A_n = \left(x_1 + x_2 + \cdots + x_n\right) + j\left(y_1 + y_2 + \cdots + y_n\right)$$

3. Products and Ratios

$$A\cdot B = \left(|A|e^{j\alpha}\right)\left(|B|e^{j\beta}\right) = |A||B|e^{j(\alpha+\beta)} \qquad A/B = \left(|A|e^{j\alpha}\right)/\left(|B|e^{j\beta}\right) = \left(|A|/|B|\right)e^{j(\alpha-\beta)}$$