Introductory Motion and Control
PID Compensators

References: R. Dorf and R. Bishop, *Modern Control Systems*, Prentice-Hall, 2001.
B. Kuo, *Automatic Control Systems*, Prentice-Hall, 1995.

<u>**PD Compensators**</u> (phase-lead type compensator)

<u>Form</u>

$$G_c(s) = K_1 + K_3 s = K_3\left(s + (K_1/K_3)\right) = K_3\left(s + a\right)$$

<u>Parameters</u>

K_3 and location of the compensator ***zero*** at $s = -K_1/K_3$

<u>Effects of PD Compensation</u>

- Compensator is anticipatory; it responds to system error and its derivative
- Phase lead is provided starting one decade (in frequency) below the zero
- Generally, increases damping and reduces %OS
- Generally, reduces rise and settling times
- Increases bandwidth
- Increases phase and gain margins
- May render a system susceptible to high frequency noise
- Acts as a high-pass filter

<u>Design Strategy</u>

Frequency Domain:

Place the ***zero*** of the compensator so the phase margin of the closed-loop system reaches its target value. Adjust gain for error control.

Root Locus (*s*-plane):

Place the ***zero*** in a location to attract poles into the target region, and then identify K_3 associated with poles in target region.

PI Compensators (phase-lag type compensator)

Form

$$G_c(s) = K_1 + K_2/s = \frac{K_1\left(s+(K_2/K_1)\right)}{s} = \frac{K_1\left(s+a\right)}{s}$$

Parameters

K_1 and the location of the compensator ***zero*** at $s = -K_2/K_1$

Effects of PI Compensation

- Compensator ***increases the system type*** by one, which helps with error control.
- ***Increases phase-lag*** at low frequencies.
- Generally, ***increases damping***, rise times, and settling times and reduces overshoot.
- ***Decreases bandwidth***.
- ***Not sensitive*** to high frequency ***noise***.
- Acts as a ***low-pass filter***.

Design Strategy

Frequency Domain:

Select the ***zero*** of the compensator as low as possible, so the phase margin of the closed-loop system is not lowered beyond specifications. Adjust gain for error control.

1) Adjust the gain of the uncompensated system for error control.
2) Find the frequency on the Bode diagram where the phase margin requirement is satisfied (assuming that is the zero-dB crossover frequency). Define this frequency as ω_c. Place the ***zero*** at least one decade below this frequency.
3) Set $-20\log(K_1) =$ gain at ω_c. That is, set $K_1 = 10^{-(\text{gain at } \omega_c)/20}$.
4) Check performance and error control. Adjust location of the ***zero*** as necessary.

Root Locus (*s*-plane):

Select the location of the ***zero*** to be relatively close to the origin and away from the most significant poles of the system. One branch of the root locus will run from the ***pole at the origin to the zero***. If the gain is selected so the pole from this branch is close to the zero of the compensator, these two will effectively cancel each other in the closed loop transfer function.

<u>PID Compensators</u> (lead-lag type compensator)

<u>Form</u>

$$G_c(s) = K_1 + K_2/s + K_3 s = \frac{K_3\left(s^2 + (K_1/K_3)s + (K_2/K_3)\right)}{s} = \frac{K(s+a)(s+b)}{s}$$

<u>Parameters</u>

K_3 and the location of the ***two compensator zeros***. The parameters a and b may be ***real*** or they may be ***complex conjugates***.

<u>Effects of PID Compensation</u>

- ***Combined effects*** of PI and PD compensation.
- ***Cascade*** of a PI and PD compensator.

<u>Design Strategy</u>

Frequency Domain:

The PID compensator is observed to be a product of PI and PD compensators. That is, the PID compensator is a PI compensator in series with a PD compensator. So, one approach to the design of the PID compensator is to design the PI and PD parts sequentially using the procedures described above.

Root Locus (s-plane):

Select the locations of the zeros of the PID compensator to provide favorable changes to the root locus diagram. Determine values of gain K for roots to be in target locations. Check for performance and error control.