

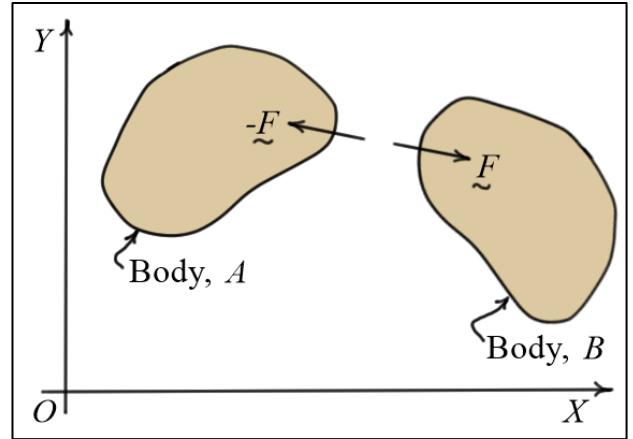
Elementary Dynamics

Conservation of Momentum and Impact for Rigid Bodies in Two Dimensions

Conservation of Linear Momentum

If the *net impulse* on a *rigid body* is *zero* over the time interval $t_1 \rightarrow t_2$, the *linear momentum* of the *body* is *conserved* (i.e. the mass center moves with constant velocity). If the *net impulse* on a *system* of *rigid bodies* is *zero* over the time interval $t_1 \rightarrow t_2$, the *linear momentum* of the *system* of bodies is *conserved*.

For example, consider two *colliding bodies*. If the *impulsive* forces \tilde{F} and $-\tilde{F}$ act over a *short* time interval $t_1 \rightarrow t_2$, the principle of linear impulse and momentum can be applied to each body over that interval. Because the impulses of \tilde{F} and $-\tilde{F}$ are *equal* and *opposite*, the *linear momentum* of the *system* of the two bodies is *conserved*. That is,



$$(m_A \mathbf{v}_A)_1 + (m_B \mathbf{v}_B)_1 = (m_A \mathbf{v}_A)_2 + (m_B \mathbf{v}_B)_2$$

Conservation of Angular Momentum

If the *net angular impulse* on a *rigid body* is *zero* over the time interval $t_1 \rightarrow t_2$, the *angular momentum* of the *body* is *conserved* (i.e. the body rotates with constant angular velocity). If the *net angular impulse* on a *system* of *rigid bodies* is *zero* over the time interval $t_1 \rightarrow t_2$, the *angular momentum* of the *system* of bodies is *conserved*. For example, consider again two colliding bodies. The *net angular impulse* of the two *impulsive* forces \tilde{F} and $-\tilde{F}$ about the *fixed point* O is zero, so the *angular momentum* of the *system* about O is *conserved* during the *impact*. That is,

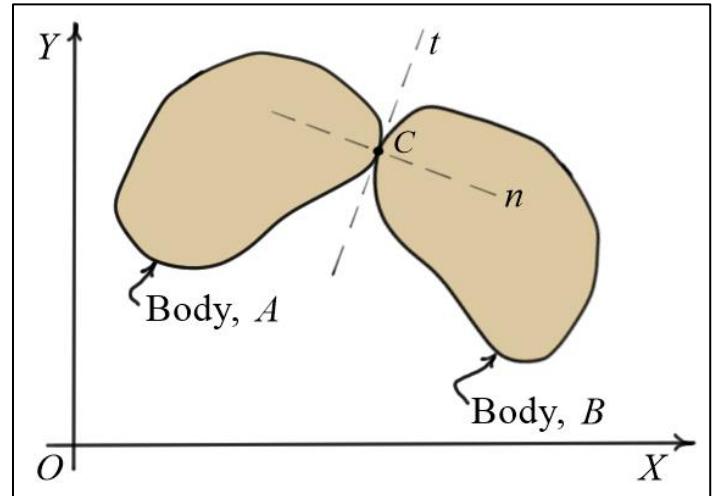
$$(\tilde{H}_O)_{A1} + (\tilde{H}_O)_{B1} = (\tilde{H}_O)_{A2} + (\tilde{H}_O)_{B2}$$

Recall the angular momentum of a body about a fixed-point O is calculated as follows.

$$\tilde{H}_O = I_G \varphi + (\mathbf{r}_G \times m \mathbf{v}_G)$$

Coefficient of Restitution

Consider two colliding bodies A and B . At the **contact point** C , the directions n and t are **normal** and **tangent** to the colliding surfaces. If the **friction forces** resulting from the impact are **negligible**, it can be shown that the **relative velocities** of the points of contact on the two bodies in the n -direction can be related through e the coefficient of restitution as follows:



$$e = \frac{(v_{CB})_{n2} - (v_{CA})_{n2}}{(v_{CA})_{n1} - (v_{CB})_{n1}}$$

Here, $(v_{CA})_{n1}$ and $(v_{CB})_{n1}$ represent the **velocities** of the **contact points** on bodies A and B in the n -direction **before impact** (time, t_1), and $(v_{CA})_{n2}$ and $(v_{CB})_{n2}$ represent the **velocities** of the **contact points** on bodies A and B in the n -direction **after impact** (time, t_2).