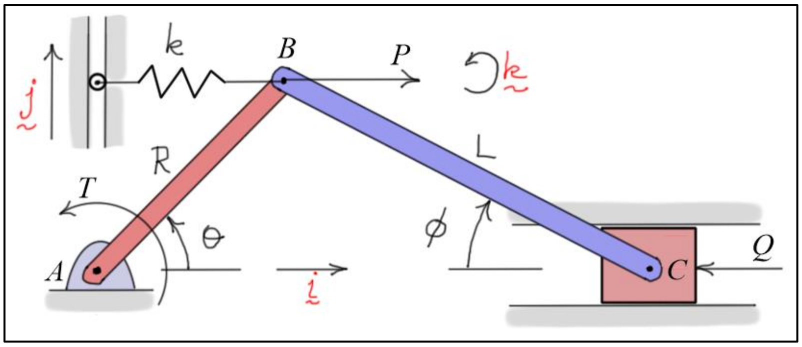
Intermediate Dynamics
Principle of Virtual Work – Example



The figure shows a ***slider crank mechanism*** with zero offset under the action of an ***external torque*** T acting on link AB, ***external forces*** P and Q acting at B and C, and a ***linear spring*** attached at B.

Problem: ***Given*** values for T and P and that the spring has unstretched length ℓ_u, ***find*** the force Q required to hold the mechanism in ***equilibrium*** at some angle θ.

Solution: (using θ as the ***generalized coordinate***)

Generalized Force (using partial velocities from previous notes)

The ***generalized force*** associated with θ is: $\boxed{F_\theta = (F_\theta)_P + (F_\theta)_Q + (F_\theta)_{spring} + (F_\theta)_T}$

- $(F_\theta)_P = (P\underset{\sim}{i}) \cdot \left(\dfrac{\partial \underset{\sim}{v}_B}{\partial \dot{\theta}} \right) = (P\underset{\sim}{i}) \cdot \dfrac{\partial}{\partial \dot{\theta}} \left(R\dot{\theta}(-S_\theta \underset{\sim}{i} + C_\theta \underset{\sim}{j}) \right) = (P\underset{\sim}{i}) \cdot \left(R(-S_\theta \underset{\sim}{i} + C_\theta \underset{\sim}{j}) \right) = \boxed{-PRS_\theta}$
- $(F_\theta)_{spring} = (-f_{sp}\underset{\sim}{i}) \cdot \left(\dfrac{\partial \underset{\sim}{v}_B}{\partial \dot{\theta}} \right) = (-f_{sp}\underset{\sim}{i}) \cdot \left(R(-S_\theta \underset{\sim}{i} + C_\theta \underset{\sim}{j}) \right) = f_{sp} RS_\theta = \boxed{k(RC_\theta - \ell_u)RS_\theta}$
- $(F_\theta)_Q = (-Q\underset{\sim}{i}) \cdot \left(\dfrac{\partial \underset{\sim}{v}_C}{\partial \dot{\theta}} \right) = (-Q\underset{\sim}{i}) \cdot \left[-R\left(S_\theta + C_\theta S_\phi / C_\phi \right) \underset{\sim}{i} \right] = \boxed{QR\left(S_\theta + C_\theta S_\phi / C_\phi \right)}$
- $(F_\theta)_T = (T\underset{\sim}{k}) \cdot \left(\dfrac{\partial \underset{\sim}{\omega}_{AB}}{\partial \dot{\theta}} \right) = T\underset{\sim}{k} \cdot \underset{\sim}{k} = \boxed{T}$

Principle of Virtual Work

Applying the ***principle of virtual work*** and solving for the force Q gives

$$F_\theta = 0 = -PRS_\theta + k(RC_\theta - \ell_u)RS_\theta + QR\left(S_\theta + C_\theta S_\phi / C_\phi \right) + T$$

$$\Rightarrow \boxed{Q = \frac{PRS_\theta - k(RC_\theta - \ell_u)RS_\theta - T}{R\left(S_\theta + C_\theta S_\phi / C_\phi \right)}}$$

Notes:

1. The ***pin forces*** at *A*, *B*, and *C* and the ***normal force*** at *C* are inactive (have no contribution).
2. The forces and torques that contribute to F_θ are ***active***.
3. There is only ***one equation*** associated with the one degree-of-freedom of the system.
4. The contribution of the spring could be calculated using ***potential energy***.