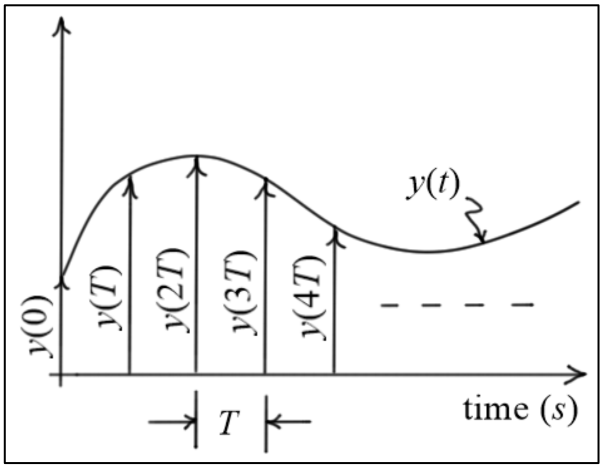
Introductory Motion and Control
Sampled Data

When a ***discrete compensator*** is used to control a ***continuous system***, some of the signals are ***continuous functions*** of time, and some are ***sampled*** at a discrete set of times. ***Sampled data*** may be generated directly by a ***digital computation*** or by ***sampling*** a ***continuous signal*** using an ***analog-to-digital*** converter. In either case, a continuous function is represented by a discrete set of values as shown below.



$$\boxed{\begin{aligned} t &= \left[0, T, 2T, \ldots, NT\right] \\ y(kT) &= \left[y(0), y(T), y(2T), \ldots, y(NT)\right] \end{aligned}}$$

Sampled Data for a Continuous Function

Two important ***characteristics*** of sampled data are the ***sample period*** T and the ***resolution***. The ***raw speed of the converter*** and the ***number of channels determine*** the ***smallest sample period*** a converter supports. In some systems, each channel of data has its own ***dedicated*** converter, while in others a single converter is ***shared*** by multiple channels by using a multiplexer. The ***resolution*** is determined by the ***number of bits*** used in the conversion process.

It is important to sample using a ***sufficiently small period*** and sufficient resolution to accurately represent the original signal. If a signal is sampled at a rate of $1/T$ samples per second, then the ***highest frequency captured*** by the sampled data is

$$\boxed{\omega_{\max} = \tfrac{1}{2}\omega_s = \tfrac{1}{2}\left(\frac{2\pi}{T}\right) \text{ (rad/sec)}}$$ … known as the ***Nyquist frequency***

Aliasing will occur if there is ***significant energy*** in the original signal above the Nyquist frequency. The signal content above the Nyquist frequency will be ***erroneously interpreted*** to be at lower frequencies.

Thus, if ***aliasing*** occurs, the ***response at frequencies above the Nyquist frequency will be missed***, and the ***response at lower frequencies will be distorted***!

Anti-aliasing (low pass) filters may be used to reduce the frequency content in the original signal above the Nyquist frequency, but ***phase effects*** will be noticeable at frequencies a ***decade*** below the ***cut-off frequency*** of the filter. These ***phase effects*** make the system appear to respond ***slower*** than it does.

It is also important to use the ***full*** voltage ***range*** of the *ADC* and *DAC* converters and use enough bits to provide a desirable ***resolution***. The ***quantization error*** for a converter using "*n*" bits in the conversion process is

$$\boxed{e_Q = \frac{V_{\max} - V_{\min}}{2^n - 1}}$$

Here, $V_{\max} - V_{\min}$ represents the ***maximum*** voltage ***range*** of the converter.