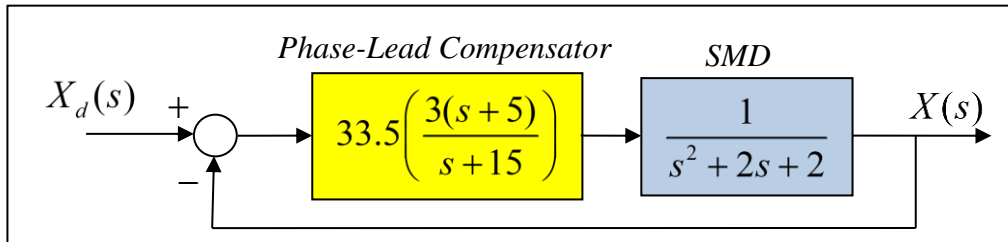# Introductory Motion and Control
# Example: Digital Phase-Lead Control of a Spring-Mass-Damper

Position control of a spring-mass-damper system using a *continuous phase-lead* compensator is shown in the diagram below.


Closed-Loop Position Control of a Spring-Mass-Damper System

In the following results, the phase-lead compensator is converted to discrete form to examine the effects that *finite sampling times* have on performance. The compensator is converted using *Tustin's approximation* at sampling times of $T = 0.001$ (sec), $T = 0.01$ (sec), $T = 0.05$ (sec), and $T = 0.1$ (sec), and the resulting approximations are implemented in the Simulink model shown below.
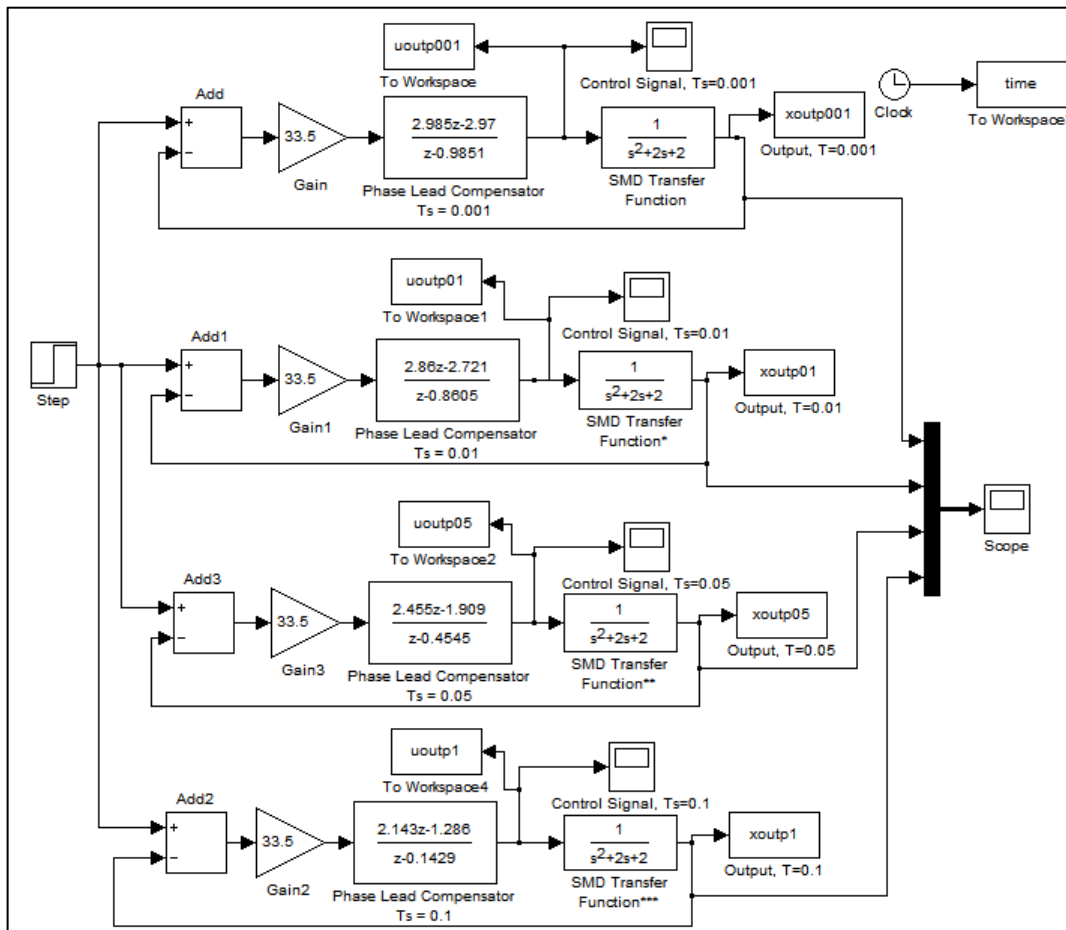

Simulink Model with Discrete Compensators

The compensator is *modeled* as if it were *implemented* using digital hardware at each of the *four sample rates*, and the *SMD* is *modeled* as a *continuous system*. The *results* for the *four different systems* are shown below. Note that the position control *degrades* as the sampling time becomes *too large*.
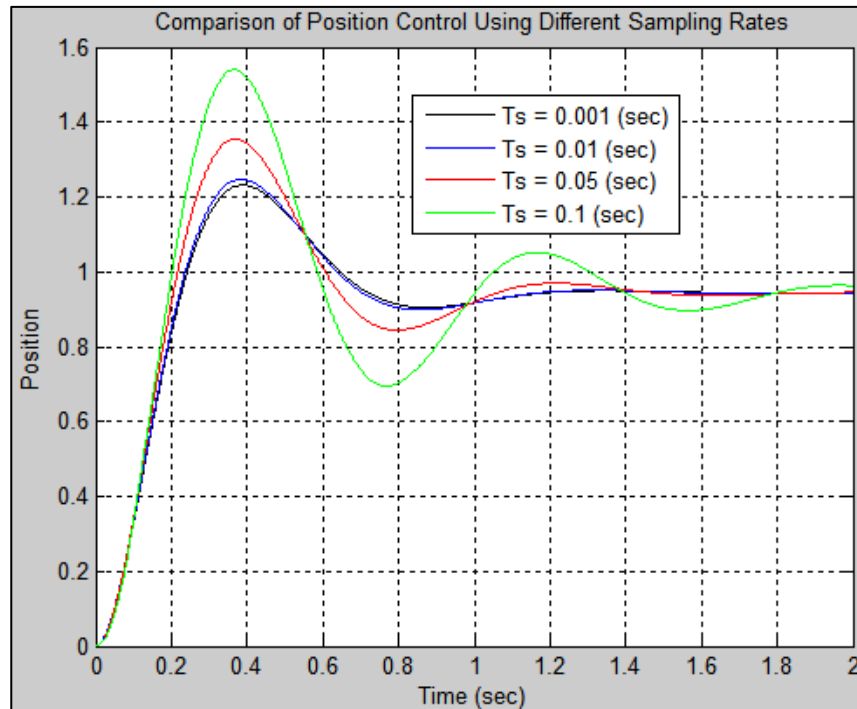

Comparison of Position Control for the Different Sample Times
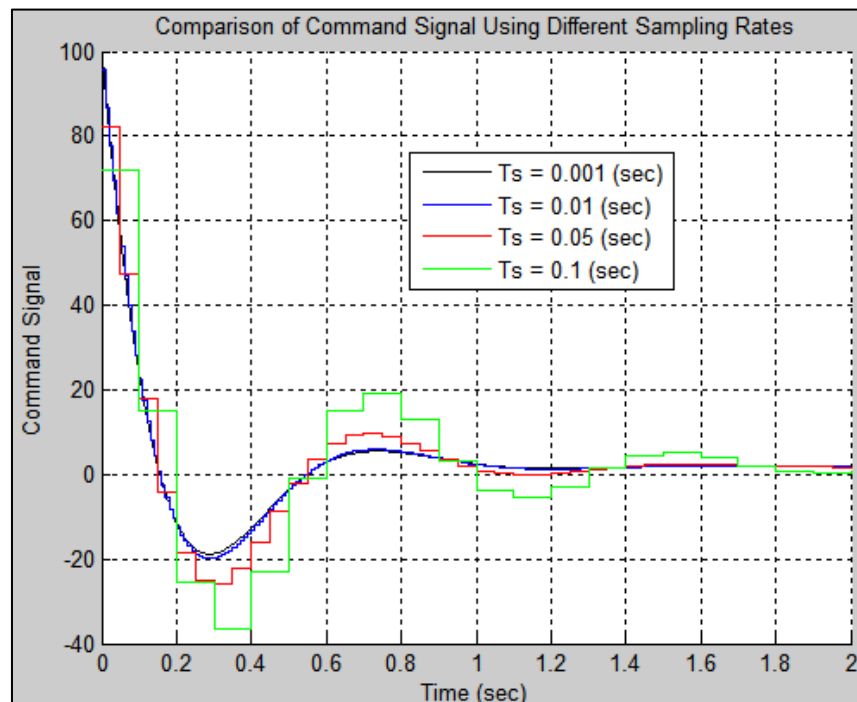
Comparison of Control Signals for the Different Sample Times

In general, *reasonable results* are expected when the sample rate is *at least 20 times* the *closed loop system bandwidth*. The Bode diagram of the closed loop system is shown below, indicating a *bandwidth* of approximately $10 \ (\text{rad/s}) = 1.6 \ (\text{Hz})$. This result suggests that the sample rate for this system should be at least $f = 20(1.6) \approx 32 \ (\text{Hz})$. This correlates to *sample times* of $T \leq 0.03 \ (\text{sec})$. This is consistent with the results shown above.
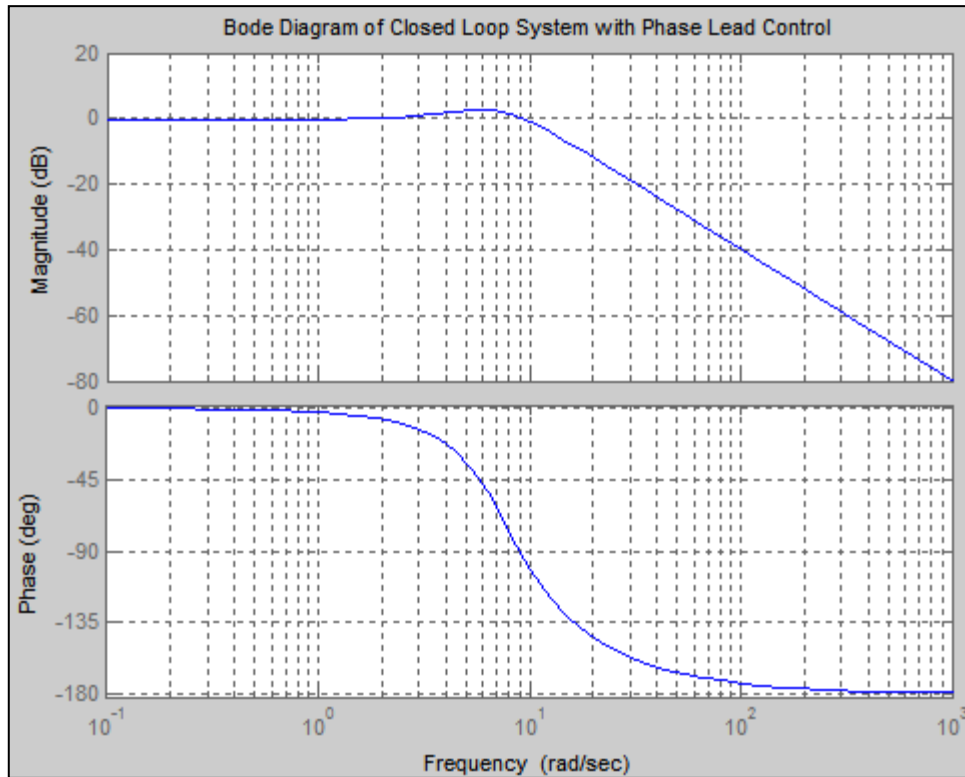
Bode Diagram of the Closed-Loop System with Phase-Lead Control