Intermediate Dynamics
Exercises #8b Answers

1. Lagrangian:

$$L = \left(\tfrac{1}{2} m_P \ell^2 S_\theta^2\right)\dot{\theta}^2 + \left(\tfrac{1}{6} m \ell^2\right)\dot{\theta}^2 - m_P\, g\, \ell\, C_\theta - \tfrac{1}{2} m\, g\, \ell\, C_\theta - \tfrac{1}{2} k\, \ell^2 S_\theta^2$$

Differential Equation of Motion:

$$\left[\tfrac{1}{3} m \ell^2 + m_P \ell^2 S_\theta^2\right]\ddot{\theta} + \left(m_P \ell^2 S_\theta C_\theta\right)\dot{\theta}^2 + \left(c\, \ell^2 C_\theta^2\right)\dot{\theta} + k\, \ell^2 S_\theta C_\theta - \left[m_P g + \tfrac{1}{2} m\, g + F(t)\right]\ell\, S_\theta = 0$$

2. Lagrangian:

$$L = \tfrac{1}{2}\left(m_1 + m_2\right)\dot{x}^2 + \left(\tfrac{1}{6} m_2 \ell^2\right)\dot{\theta}^2 + \left(\tfrac{1}{2} m_2 \ell\, C_\theta\right)\dot{x}\,\dot{\theta} - \tfrac{1}{2} k\, x^2 + \tfrac{1}{2} m_2\, g\, \ell\, C_\theta$$

Differential Equations of Motion:

$$\left(m_1 + m_2\right)\ddot{x} + \left(\tfrac{1}{2} m_2\, \ell\, C_\theta\right)\ddot{\theta} - \left(\tfrac{1}{2} m_2 \ell\, S_\theta\right)\dot{\theta}^2 + c\,\dot{x} + k\, x = F(t)$$

$$\left(\tfrac{1}{3} m_2\, \ell^2\right)\ddot{\theta} + \left(\tfrac{1}{2} m_2\, \ell\, C_\theta\right)\ddot{x} + \tfrac{1}{2} m_2\, g\, \ell\, S_\theta = 0$$

3. Kinetic Energy:

$$K = \tfrac{1}{4} m_S\, r^2\, \dot{\phi}^2 + \tfrac{1}{24} m \ell^2 \left(\dot{\theta}^2 + \dot{\phi}^2 S_\theta^2\right)$$

Differential Equations of Motion:

$$\left[\tfrac{1}{2} m_S r^2 + \tfrac{1}{12} m \ell^2 S_\theta^2\right]\ddot{\phi} + \left(\tfrac{1}{6} m \ell^2 S_\theta C_\theta\right)\dot{\theta}\,\dot{\phi} = M_\phi$$

$$\left(\tfrac{1}{12} m \ell^2\right)\ddot{\theta} - \left(\tfrac{1}{12} m \ell^2 S_\theta C_\theta\right)\dot{\phi}^2 = M_\theta$$

4. Lagrangian:

$$L = \left[\tfrac{1}{4} m_d R^2 + \tfrac{1}{2} m\left(b + \tfrac{1}{2}\ell\, S_\theta\right)^2 + \tfrac{1}{24} m \ell^2 S_\theta^2\right]\dot{\phi}^2 + \left(\tfrac{1}{6} m \ell^2\right)\dot{\theta}^2 - \tfrac{1}{2} k\, \theta^2 + \tfrac{1}{2} m\, g\, \ell\, C_\theta$$

Differential Equations of Motion:

$$\left[\tfrac{1}{2} m_d R^2 + m(b + \tfrac{1}{2}\ell\, S_\theta)^2 + \tfrac{1}{12} m \ell^2 S_\theta^2\right]\ddot{\phi} + \left[m\, b\, \ell\, C_\theta + \tfrac{2}{3} m \ell^2 S_\theta C_\theta\right]\dot{\theta}\,\dot{\phi} = M_\phi$$

$$\left(\tfrac{1}{3} m \ell^2\right)\ddot{\theta} - \left[\tfrac{1}{2} m\, b\, \ell\, C_\theta + \tfrac{1}{3} m \ell^2 S_\theta C_\theta\right]\dot{\phi}^2 + c\,\dot{\theta} + k\,\theta + \tfrac{1}{2} m\, g\, \ell\, S_\theta = M_\theta$$