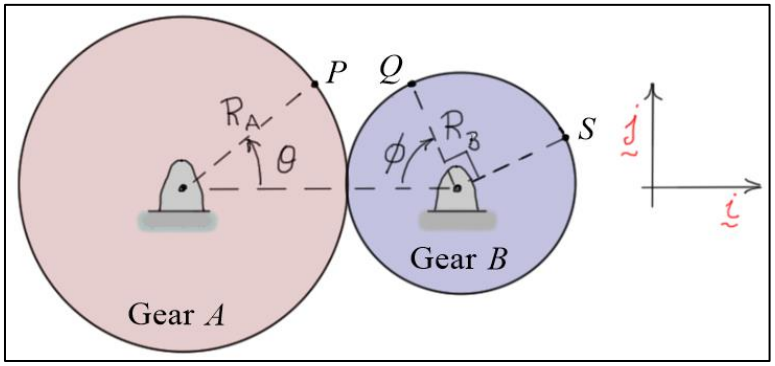
Elementary Dynamics Example #12: (2D Motion, Relative Motion)

Given: $R_A = 3$ (in), $R_B = 2$ (in)

$\omega_A = \dot{\theta} = 1000$ (rpm) … constant

$\omega_B = \dot{\phi} = 1500$ (rpm) … constant

- when $\theta = \phi = 0$, P and Q are touching, and S is directly above the support.



Find: $\underset{\sim}{v}_{Q/P}$, $\underset{\sim}{a}_{Q/P}$, $\underset{\sim}{v}_{S/P}$, $\underset{\sim}{a}_{S/P}$ at $\boxed{\theta = \phi = 0}$

Solution:

$$\underset{\sim}{v}_P = R_A\dot{\theta}\,\underset{\sim}{j} = \left(\tfrac{3}{12}\right)\left(1000\left(\tfrac{2\pi}{60}\right)\right)\underset{\sim}{j} \approx 26.1799\,\underset{\sim}{j} \approx 26.2\,\underset{\sim}{j}\ \text{(ft/s)}$$

$$\underset{\sim}{v}_Q = R_B\dot{\phi}\,\underset{\sim}{j} = \left(\tfrac{2}{12}\right)\left(1500\left(\tfrac{2\pi}{60}\right)\right)\underset{\sim}{j} \approx 26.1799\,\underset{\sim}{j} \approx 26.2\,\underset{\sim}{j}\ \text{(ft/s)}$$

$$\boxed{\underset{\sim}{v}_{Q/P} = \underset{\sim}{v}_Q - \underset{\sim}{v}_P = 0}$$

Note: for gears in contact, $\boxed{R_A\dot{\theta} = R_B\dot{\phi}}$ where R_A and R_B represent the ***pitch radii*** of the two gears.

$$\underset{\sim}{a}_P = -\left(R_A\dot{\theta}^2\right)\underset{\sim}{i} = -\left(\tfrac{3}{12}\right)\left(1000\left(\tfrac{2\pi}{60}\right)\right)^2\underset{\sim}{i} \approx -2741.56\,\underset{\sim}{i}\ \text{(ft/s}^2\text{)}$$

$$\underset{\sim}{a}_Q = \left(R_B\dot{\phi}^2\right)\underset{\sim}{i} = \left(\tfrac{2}{12}\right)\left(1500\left(\tfrac{2\pi}{60}\right)\right)^2\underset{\sim}{i} \approx 4112.34\,\underset{\sim}{i}\ \text{(ft/s}^2\text{)}$$

$$\boxed{\underset{\sim}{a}_{Q/P} = \underset{\sim}{a}_Q - \underset{\sim}{a}_P \approx 6850\,\underset{\sim}{i}\ \text{(ft/s}^2\text{)}}$$

So, the ***relative acceleration*** of the two contacting points is ***not zero***.

$$\underset{\sim}{v}_S = R_B\dot{\phi}\,\underset{\sim}{i} = \left(\tfrac{2}{12}\right)\left(1500\left(\tfrac{2\pi}{60}\right)\right)\underset{\sim}{i} \approx 26.1799\,\underset{\sim}{i} \approx 26.2\,\underset{\sim}{i}\ \text{(ft/s)}$$

$$\boxed{\underset{\sim}{v}_{S/P} = \underset{\sim}{v}_S - \underset{\sim}{v}_P = 26.2\,\underset{\sim}{i} - 26.2\,\underset{\sim}{j}\ \text{(ft/s)}}$$

45°

37.0 (ft/s)

Finally,

$$\underset{\sim}{a}_S = -\left(R_B\dot{\phi}^2\right)\underset{\sim}{j} = -\left(\tfrac{2}{12}\right)\left(1500\left(\tfrac{2\pi}{60}\right)\right)^2\underset{\sim}{j} \approx -4112.34\,\underset{\sim}{j}\ \text{(ft/s}^2\text{)}$$

$$\boxed{\underset{\sim}{a}_{S/P} = \underset{\sim}{a}_S - \underset{\sim}{a}_P \approx 2740\,\underset{\sim}{i} - 4110\,\underset{\sim}{j}\ \text{(ft/s}^2\text{)}}$$

56.3°

4940 (ft/s^2)