

Elementary Dynamics Example #15: (Newton's Laws, Normal & Tangential Components)

Given: $mg = 10$ (lb), $T_{\max} = 20$ (lb), $L = 5$ (ft)

v is constant so m has **circular motion**

Find: v_{\max} the velocity of m that makes the tension

$T = T_{\max}$ and the corresponding angle constant

angle θ

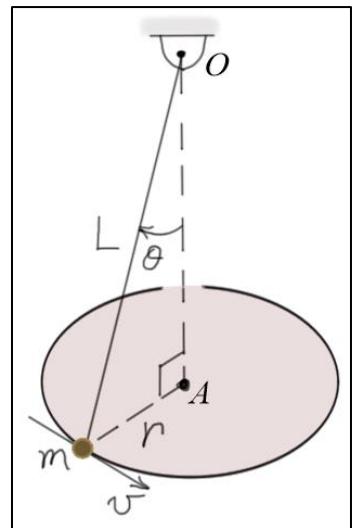
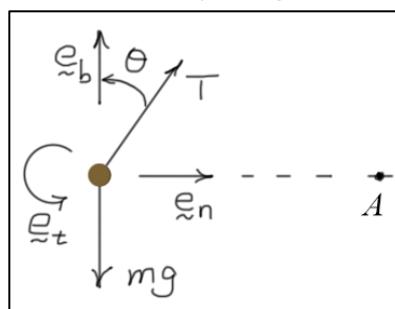
Solution: with $T = T_{\max} = 20$ (lb)

$$\sum F_t = 0 \quad (\text{no forces in that direction})$$

$$\uparrow \sum F_b = T \cos(\theta) - mg = 0$$

$$\rightarrow \sum F_n = T \sin(\theta) = m \left(\frac{v^2}{r} \right)$$

Free body diagram



Setting $T = T_{\max} = 20$ (lb), we find $\cos(\theta) = \frac{mg}{T_{\max}} = \frac{10}{20}$ $\Rightarrow \theta = \cos^{-1}\left(\frac{1}{2}\right) = 60$ (deg)

Using this value of θ gives

$$m \left(\frac{v_{\max}^2}{r} \right) = m \left(\frac{v_{\max}^2}{L \sin(\theta)} \right) = T \sin(\theta) \Rightarrow v_{\max} = \left(\sqrt{\frac{TL}{m}} \right) \sin(\theta) \approx 15.5403 \approx 15.5 \text{ (ft/s)}$$