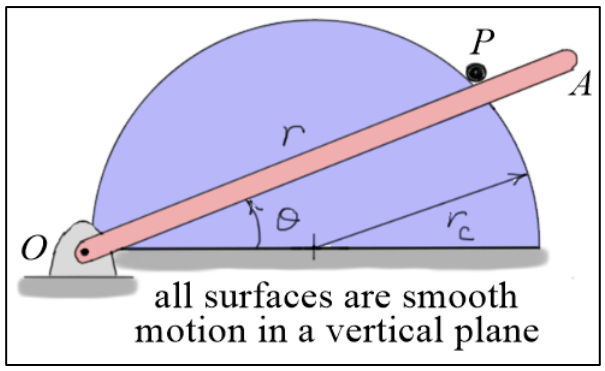
Elementary Dynamics Example #18: (Newton's Laws, Radial & Transverse Components)

Given: $W_P = W = 0.5$ (lb), size of P is negligible
$r(\theta) = 2 r_C \cos(\theta)$ (ft), $r_C = 0.4$ (ft)
@ $\theta = 30$ (deg), $\dot{\theta} = 0.4$ (r/s), $\ddot{\theta} = 0.8$ (r/s^2)
thickness of bar OA is negligible

Find: F_{OA} the force bar OA exerts on P
F_S the force the fixed surface exerts on P



Solution:

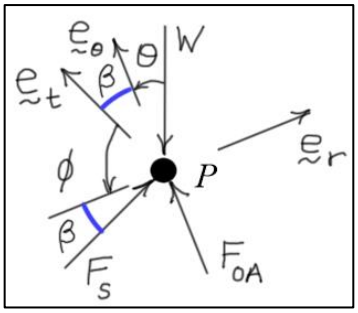
Kinematics:

$$r(30) = 2(0.4)\cos(30) \approx 0.69282 \text{ (ft)}, \quad v_\theta = r\dot{\theta} \approx 0.277128 \text{ (ft/s)}$$

$$v_r(30) = (\dot{r})_{\theta=30^o} = \left(-0.8\dot{\theta}\sin(\theta)\right)_{\theta=30^o} \approx -0.16 \text{ (ft/s)}$$

$$\phi = \tan^{-1}\left(\frac{v_\theta}{|v_r|}\right) = \tan^{-1}\left(\frac{0.277128}{0.16}\right) = 60 \text{ (deg)} \qquad \beta = 90 - \phi = 30 \text{ (deg)}$$

Free body diagram



$$(\ddot{r})_{\theta=30^o} = \left(\frac{d\dot{r}}{dt}\right)_{\theta=30^o} = \left(-0.8\left(\ddot{\theta}\sin(\theta) + \dot{\theta}^2\cos(\theta)\right)\right)_{\theta=30^o} \approx -0.430851 \text{ (ft/s}^2\text{)}$$

$$a_r = \ddot{r} - r\dot{\theta}^2 \approx -0.541703 \text{ (ft/s}^2\text{)}, \quad a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} \approx 0.426256 \text{ (ft/s}^2\text{)}$$

Kinetics:

$$\nearrow \sum F_r = F_s\cos(30) - W\sin(\theta) = \left(\frac{W}{g}\right)a_r$$

$$\Rightarrow F_s = \left(W\sin(\theta) + \left(\frac{W}{g}\right)a_r\right)\Big/\cos(30) \approx 0.278962 \approx 0.279 \text{ (lb)}$$

$$\nwarrow \sum F_\theta = F_{OA} + F_s\sin(30) - W\cos(\theta) = \left(\frac{W}{g}\right)a_\theta$$

$$\Rightarrow F_{OA} = -F_s\sin(30) + W\cos(\theta) + \left(\frac{W}{g}\right)a_\theta \approx 0.30015 \approx 0.300 \text{ (lb)}$$