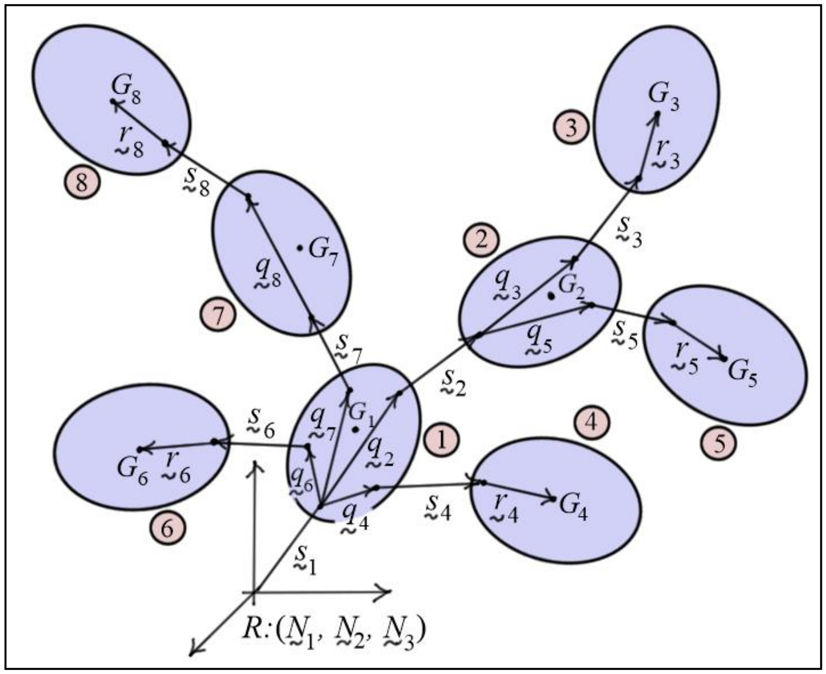
Multibody Dynamics
Body-Connection Array
(Reference: R. L. Huston, *Multibody Dynamics*, Butterworth-Heinemann, 1990.)



- The development of the kinematic and dynamic equations of motion of a multibody system can be structured using a ***body-connection array***.
- Consider, for example, the system shown in the diagram. The system consists of ***eight bodies*** connected to form an ***open-tree system***. An open-tree system is one whose branches ***do not*** connect with each other.
- If two or more branches of a system connect, the system is said to have ***closed kinematic chains***.
- The bodies of the system can be ***numbered*** by first choosing a ***reference body*** for the system and naming it B_1. Then, name the remaining bodies in ***ascending progression*** away from B_1 through the branches of the system as shown in the diagram.
- The body-connection array $\mathfrak{L}(k)$ is formed by ***identifying***, for each body, its ***adjoining lower-numbered body*** (*LNB*) that is one body ***closer*** to the reference body in the branch. For example, the *LNB* of B_8 is B_7, and the *LNB* of B_7 is B_1.
- Using this idea, the ***body-connection array*** for the system above can be defined as $\boxed{\mathfrak{L}(k) = \left(0,1,2,1,2,1,1,7\right)}$. Here, ***zero*** has been used to denote the ***fixed reference frame***.
- The body-connection array can be used to move from a body in the system back to the fixed frame by ***successive sampling*** of the array. For example, to move from body B_5 to the inertial frame, note that

$$\mathfrak{L}^0(5) = 5 \quad \Rightarrow \quad B_5$$
$$\mathfrak{L}^1(5) = \mathfrak{L}(5) = 2 \quad \Rightarrow \quad B_2$$

$$\mathcal{L}^2(5) = \mathcal{L}\big(\mathcal{L}(5)\big) = \mathcal{L}(2) = 1 \quad \Rightarrow \quad B_1$$
$$\mathcal{L}^3(5) = \mathcal{L}\big(\mathcal{L}^2(5)\big) = \mathcal{L}\big(\mathcal{L}\big(\mathcal{L}(5)\big)\big) = 0 \quad \Rightarrow \quad \text{fixed frame}$$

- The body connection array can be used to find u_K the ***number of bodies below*** a body in the system. This is done by finding the integer u_K such that $\mathcal{L}^{u_K}(K) = 1$. For example, from the above equations, we know that $u_5 = 2$, because $\mathcal{L}^2(5) = 1$. This means there are ***two bodies*** between body B_5 and the fixed frame.
- This idea can be used to organize the development of the equations of motion of the system. For example, the position vector of G_5 the mass-center of body B_5 can be written as

$$\boxed{\underset{\sim}{p}_5 = \underset{\sim}{s}_1 + \sum_{r=0}^{u_5-1}\left(\underset{\sim}{q}_{\mathcal{L}^r(5)} + \underset{\sim}{s}_{\mathcal{L}^r(5)}\right) + \underset{\sim}{r}_5}$$