

### Elementary Dynamics Example #27: (Conservation of Momentum, Impact)

Given:  $m_A = m_B = m$ ,  $e = 0.6$ ,

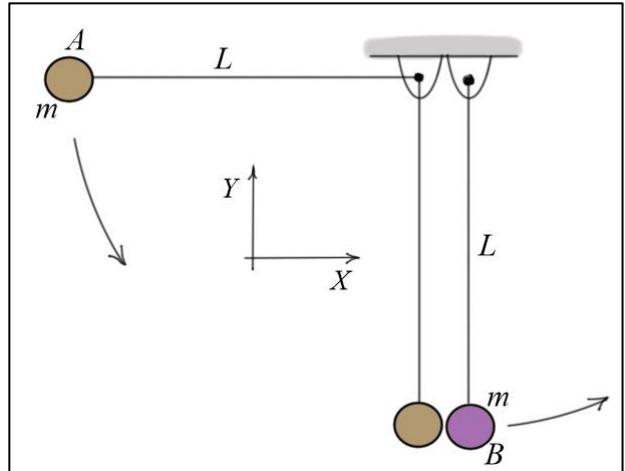
$m_A$  is released from rest at the top and strikes  $m_B$  at the bottom

Find:  $h$  the **maximum height** to which  $m_B$  swings

Solution: (*conservation of momentum and energy*)

1. Using **conservation of energy**, the velocity of  $m_A$  as it strikes  $m_B$  is found.

$$\cancel{K_1 + V_1} = K_2 + \cancel{V_2} \quad (\text{datum at the bottom})$$



So,

$$V_1 = mgL = K_2 = \frac{1}{2}m(v_{A2})^2 \Rightarrow v_{A2} = \sqrt{2gL} \quad (\text{just before impact})$$

2. Using conservation of momentum: (in  $X$  direction, 2 – just before impact, 3 – just after)

$$\cancel{m(v_{A2}) + m(v_{B2})} = \cancel{m(v_{A3})} + m(v_{B3})$$

$$\Rightarrow v_{A3} + v_{B3} = v_{A2} = \sqrt{2gL}$$

Using the impact (restitution) equation:

$$\frac{v_{A3} - v_{B3}}{\cancel{v_{B2}} - v_{A2}} = e \Rightarrow v_{A3} - v_{B3} = -e v_{A2} = -0.6\sqrt{2gL}$$

Solving the two equations simultaneously gives:

$$v_{A3} = 0.2\sqrt{2gL}, \quad v_{B3} = 0.8\sqrt{2gL}$$

3. Using conservation of energy for  $m_B$  after impact:

$$\cancel{K_3 + V_3} = \cancel{K_4} + V_4 \quad (\text{datum at the bottom})$$

So,

$$K_3 = \frac{1}{2}m(v_{B3})^2 = \cancel{\frac{1}{2}m(0.8)^2} (\cancel{gL}) = 0.64mgL = V_4 = mgh \Rightarrow h = 0.64L$$

Question: What would the result be if  $e = 1$ ?

Answer: Using the above approach, it is easy to show that, in this case,  $h = L$ . This makes sense if you recall that **kinetic energy** of the impact is **conserved** if  $e = 1$ .