

Elementary Dynamics Example #29: (Rigid Body Kinematics – Pure Rotational Motion)

Given: $r_1 = 6$ (in), $r_2 = 4$ (in), $r_3 = 2$ (in)

$\omega_1 = 1000$ (rpm) CCW

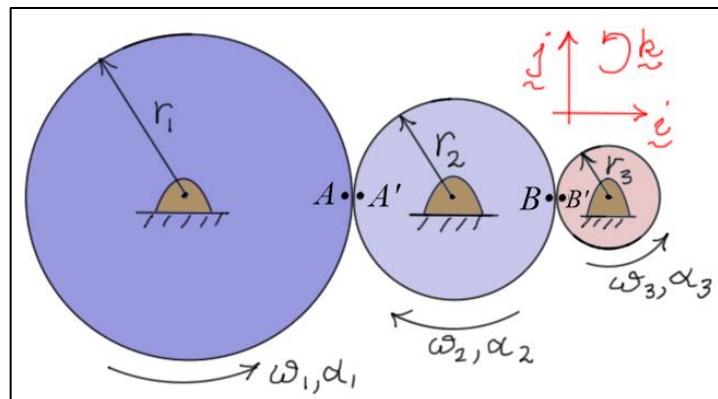
$\alpha_1 = 5$ (r/s²) CCW

- no slipping between the gears
- radii represent the pitch circles

Find: a) ω_2, ω_3

b) α_2, α_3

c) $a_A, a_{A'}$



Solution:

a) Because the gears do not slip relative to each other, $v_A = v_{A'} = r_2 \omega_2 j$ and $v_B = v_{B'} = -r_3 \omega_3 j$.

$$v_A = r_1 \omega_1 j = r_2 \omega_2 j \Rightarrow \omega_2 = \left(\frac{r_1}{r_2}\right) \omega_1 = \frac{3}{2}(1000) = 1500 \text{ (rpm) CW}$$

$$v_B = -r_2 \omega_2 j = -r_3 \omega_3 j \Rightarrow \omega_3 = \left(\frac{r_2}{r_3}\right) \omega_2 = \frac{4}{2}(1500) = 3000 \text{ (rpm) CCW}$$

Each of these rates can be expressed in radians/seconds by multiplying by $\frac{2\pi}{60}$. For example,

$$1000 \frac{\text{rev}}{\text{min}} \times \frac{1 \text{ min}}{60 \text{ sec}} \times \frac{2\pi \text{ rad}}{1 \text{ rev}} = 1000 \left(\frac{2\pi}{60}\right) \approx 104.7 \text{ (rad/s)}$$

b) Because the gears do not slip, the accelerations of the contact points **tangent** to the contacting surfaces must be equal. That is, $(a_A)_{\tan} = (a_{A'})_{\tan}$ and $(a_B)_{\tan} = (a_{B'})_{\tan}$.

$$(a_A)_{\tan} = r_1 \alpha_1 j = (a_{A'})_{\tan} = r_2 \alpha_2 j \Rightarrow \alpha_2 = \left(\frac{r_1}{r_2}\right) \alpha_1 = \frac{6}{4}(5) = 7.5 \text{ (r/s}^2\text{) CW}$$

$$(a_B)_{\tan} = -r_2 \alpha_2 j = (a_{B'})_{\tan} = -r_3 \alpha_3 j \Rightarrow \alpha_3 = \left(\frac{r_2}{r_3}\right) \alpha_2 = \frac{4}{2}(7.5) = 15 \text{ (r/s}^2\text{) CCW}$$

c) For A, $e_r = i$ and $e_{\theta} = j$. For A' , $e_r = -i$ and $e_{\theta} = j$. Using the equations for radial and transverse acceleration for constant radius motion,

$$a_A = -r_1 \omega_1^2 i + r_1 \alpha_1 j = \left(-\left(\frac{6}{12}\right)(104.72)^2\right) i + \left(\left(\frac{6}{12}\right)5\right) j \approx -5480 i + 2.5 j \text{ (ft/s}^2\text{)}$$

$$a_{A'} = r_2 \omega_2^2 i + r_2 \alpha_2 j = \left(-\left(\frac{4}{12}\right)\left(\frac{1500(2\pi)}{60}\right)^2\right) i + \left(\left(\frac{4}{12}\right)7.5\right) j \approx 8225 i + 2.5 j \text{ (ft/s}^2\text{)}$$

Note that the accelerations of A and A' are **equal** only in the **tangential** direction, j .

Their accelerations in the i direction **differ** in both **magnitude** and **sign**.