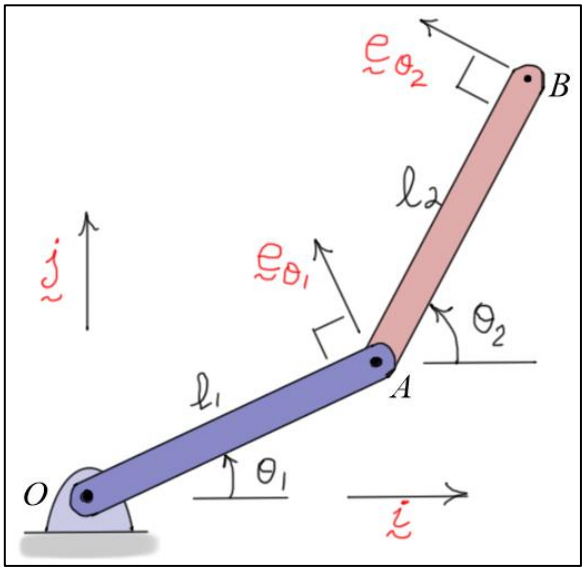
Elementary Dynamics Example #30: (Rigid Body Kinematics – Relative Velocity)

Given: $\ell_1 = \ell_2 = 0.4$ (m)

$\theta_1 = 25$ (deg), $\omega_1 = \dot{\theta}_1 = 10$ (r/s) CCW

$\theta_2 = 60$ (deg), $\omega_2 = \dot{\theta}_2 = 5$ (r/s) CCW

Find: $\underset{\sim}{v}_B$ the velocity of the endpoint B



Solution:

Using the relative velocity equation for two points fixed on a rigid body,

$$\underset{\sim}{v}_B = \underset{\sim}{v}_A + \underset{\sim}{v}_{B/A} = \underbrace{\underset{\sim}{v}_O}_{\text{zero}} + \underset{\sim}{v}_{A/O} + \underset{\sim}{v}_{B/A}$$

$$= \left[\underset{\sim}{\omega}_{OA} \times \underset{\sim}{r}_{A/O}\right] + \left[\underset{\sim}{\omega}_{AB} \times \underset{\sim}{r}_{B/A}\right]$$

$$= \left[\omega_1 \underset{\sim}{k} \times \ell_1 \left(\cos(25)\,\underset{\sim}{i} + \sin(25)\,\underset{\sim}{j}\right)\right] + \left[\omega_2 \underset{\sim}{k} \times \ell_2 \left(\cos(60)\,\underset{\sim}{i} + \sin(60)\,\underset{\sim}{j}\right)\right]$$

$$= \ell_1 \omega_1 \underbrace{\left(-\sin(25)\,\underset{\sim}{i} + \cos(25)\,\underset{\sim}{j}\right)}_{\underset{\sim}{e}_{\theta_1}} + \ell_2 \omega_2 \underbrace{\left(-\sin(60)\,\underset{\sim}{i} + \cos(60)\,\underset{\sim}{j}\right)}_{\underset{\sim}{e}_{\theta_2}}$$

$$= \left(-1.69047\,\underset{\sim}{i} + 3.62523\,\underset{\sim}{j}\right) + \left(-1.73205\,\underset{\sim}{i} + 1\,\underset{\sim}{j}\right)$$

$$\Rightarrow \boxed{\underset{\sim}{v}_B \approx -3.42\,\underset{\sim}{i} + 4.63\,\underset{\sim}{j}}$$

Notes:

1. The direction of $\underset{\sim}{v}_{A/O}$ the velocity of A with respect to O is in the $\underset{\sim}{e}_{\theta_1}$ direction which is ***perpendicular*** to link OA as shown.
2. The direction of $\underset{\sim}{v}_{B/A}$ the velocity of B with respect to A is in the $\underset{\sim}{e}_{\theta_2}$ direction which is ***perpendicular*** to link AB as shown.
3. The directions of each of these relative velocities is determined by the ***cross*** (or vector) ***product***. In each case, the direction is perpendicular to ***both*** the angular velocity vector and the relative position vector of the two points, O and A for the first link and A and B for the second link.