

Elementary Dynamics Example #31: (Rigid Body Kinematics – Relative Velocity)

Given: r, ω

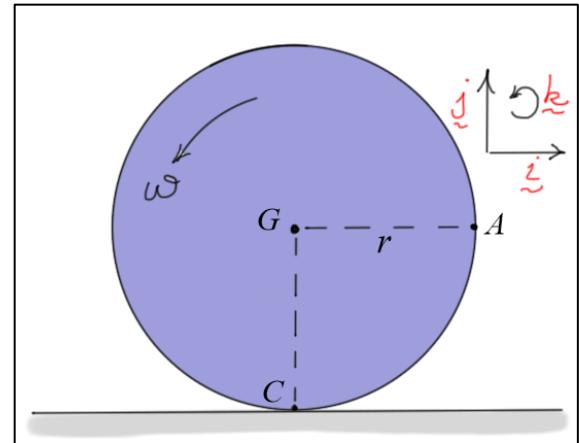
wheel rolls without slipping ($v_C = 0$).

Find: a) v_G the velocity of G

b) v_A the velocity of A

Solution:

a) Using the relative velocity equation for two points fixed on a rigid body,



$$v_G = v_C + v_{G/C} = v_C + v_{G/C} = [\omega \times r_{G/C}] = \omega \hat{k} \times r \hat{j}$$

zero

$$\Rightarrow v_G = -r\omega \hat{i}$$

b) Again, using the relative velocity equation,

$$v_A = v_C + v_{A/C} = v_C + v_{A/C} = [\omega \times r_{A/C}] = \omega \hat{k} \times (r \hat{i} + r \hat{j})$$

zero

$$\Rightarrow v_A = -r\omega \hat{i} + r\omega \hat{j}$$

or,

$$v_A = v_G + v_{A/G} = -r\omega \hat{i} + v_{A/G} = -r\omega \hat{i} + [\omega \times r_{A/G}] = -r\omega \hat{i} + [\omega \hat{k} \times r \hat{i}]$$

$$\Rightarrow v_A = -r\omega \hat{i} + r\omega \hat{j} \quad \dots \text{ same result}$$

Notes:

1. Because the wheel **rolls without slipping**, the contact point has the **same velocity** as the contacting surface which, in this case, is **zero**.
2. In the above form, the relative velocity equation can be used to relate the velocities of **any two points** on a rigid body in terms of its **angular velocity**.
3. **Velocity** is a **property** that applies to the **points** of a body, whereas **angular velocity** is a **property** that applies to an **entire body**, not the individual points of that body.