

Elementary Dynamics Example #36: (Rigid Body Kinematics – Relative Acceleration)

Given: ω, α, r, R

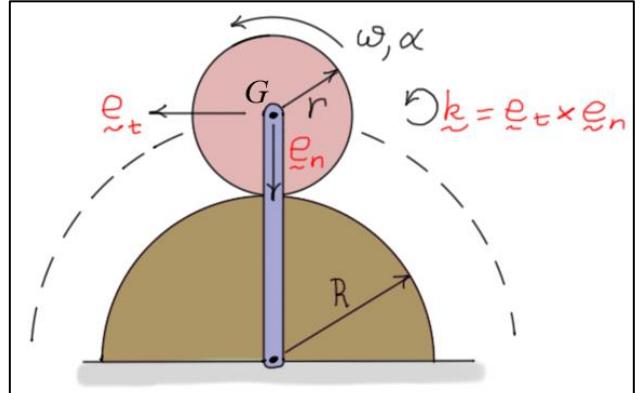
no slip between the fixed circular surface (sun gear) and the planetary gear

C is the contact point on the planetary gear

Find: \ddot{a}_G, \ddot{a}_C

Solution:

From a previous example on relative velocity,



$$\dot{v}_G = v e_t = r \omega e_t$$

Using normal and tangential components, the acceleration of G can be written as

$$\ddot{a}_G = \dot{v} e_t + \left(\frac{v^2}{\rho} \right) e_n = r \dot{\omega} e_t + \left(\frac{(r\omega)^2}{R+r} \right) e_n = r \alpha e_t + \left(\frac{(r\omega)^2}{R+r} \right) e_n$$

Using the relative acceleration equation, the acceleration of C can be written as

$$\ddot{a}_C = \ddot{a}_G + \ddot{a}_{C/G} \quad (C \text{ and } G \text{ are two points on the planetary gear})$$

where

$$\ddot{a}_{C/G} = [\alpha \dot{k} \times \dot{r}_{C/G}] - \omega^2 \dot{r}_{C/G} = [\alpha \dot{k} \times r e_n] - \omega^2 (r e_n) = -r \alpha e_t - r \omega^2 e_n$$

Substituting the two results into the relative acceleration equation gives

$$\begin{aligned} \ddot{a}_C &= (r \alpha - r \alpha) e_t + \left(\frac{(r\omega)^2}{R+r} - r \omega^2 \right) e_n = \left(\frac{r^2 \omega^2 - r \omega^2 (R+r)}{R+r} \right) e_n \\ &= \left(\frac{r^2 \omega^2 - r^2 \omega^2 - r R \omega^2}{R+r} \right) e_n \quad \Rightarrow \boxed{\ddot{a}_C = - \left(\frac{r R}{r+R} \right) \omega^2 e_n} \end{aligned}$$

As expected, the acceleration of C the contact point on the moving body (the planetary gear) is normal (perpendicular) to the contacting surfaces.