

## Elementary Dynamics Example #46: (Rigid Body Kinetics – Conservation of Energy)

Given:  $AB$  and  $BC$  are slender rods,  $\ell = 3$  (ft)

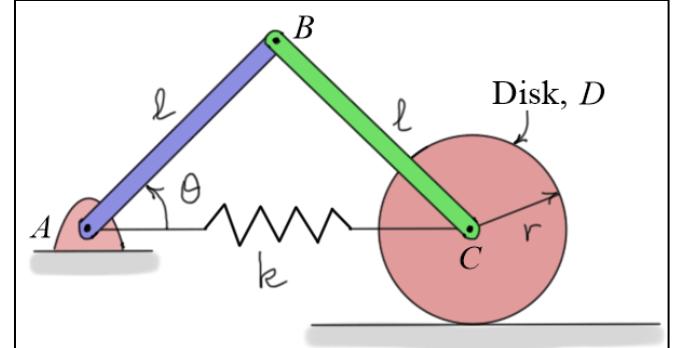
$$W_{AB} = W_{BC} = W = 15 \text{ (lb)}, W_D = 20 \text{ (lb)}$$

$$k = 4 \text{ (lb/ft)}$$

spring is unstretched at  $\theta = 45$  (deg)

released from rest at  $\theta = 45$  (deg)

Disk rolls without slipping,  $r = 1$  (ft)



Find:  $\omega_{AB}$  the angular velocity of  $AB$  at  $\theta = 0$  (deg)

Solution:

Using the **conservation of energy** on the system (state 1:  $\theta = 45$  (deg); state 2:  $\theta = 0$  (deg))

$$\boxed{\begin{matrix} K_1 + V_1 = K_2 + V_2 \\ \text{zero} \end{matrix}} \quad (\text{released from rest})$$

Defining a datum at the horizontal line through  $A$  and  $C$  gives

$$V_1 = (V_1)_{\text{gravity}} + \underbrace{(V_1)_{\text{spring}}}_{\text{zero}} = 2\left(W \frac{\ell}{2} \sin(45)\right) \approx 31.8198 \text{ (ft-lb)}$$

$$V_2 = \underbrace{(V_2)_{\text{gravity}}}_{\text{zero}} + (V_2)_{\text{spring}} = \frac{1}{2}k(2\ell - 2\ell \cos(45))^2 = 2k\ell^2(1 - \cos(45))^2 \approx 6.17662 \text{ (ft-lb)}$$

In state 2, the disk has **zero velocity**,  $AB$  is rotating about  $A$ , and the **instantaneous center** of  $BC$  is at  $C$ . Also, due to the **symmetry** of the mechanism, the angular velocities of  $AB$  and  $BC$  are **equal** (but in opposite directions).

$$\begin{aligned} K_2 &= (K_2)_{\text{disk}} + (K_2)_{AB} + (K_2)_{BC} = \underbrace{0}_{(v_C)_2 = 0} + \frac{1}{2}I_A\omega_{AB}^2 + \frac{1}{2}I_C\omega_{BC}^2 \\ &= \frac{1}{2}\left(\frac{1}{3}\left(\frac{W}{g}\right)\ell^2\right)\omega_{AB}^2 + \frac{1}{2}\left(\frac{1}{3}\left(\frac{W}{g}\right)\ell^2\right)\omega_{AB}^2 = \frac{1}{3}\left(\frac{W}{g}\right)\ell^2\omega_{AB}^2 \approx 1.39752\omega_{AB}^2 \end{aligned}$$

Substituting into the conservation of energy equation gives

$$31.8198 = 6.17662 + 1.39752\omega_{AB}^2 \Rightarrow \omega_{AB} = \sqrt{\frac{31.8198 - 6.17662}{1.39752}} \approx 4.28 \text{ (rad/s)}$$

The motion of  $AB$  is **clockwise**, and the motion of  $BC$  is **counter-clockwise**.

Question: What other forces (besides the spring and gravity) affect the motion of this system?

Why is the work done by these forces on the system equal to zero?