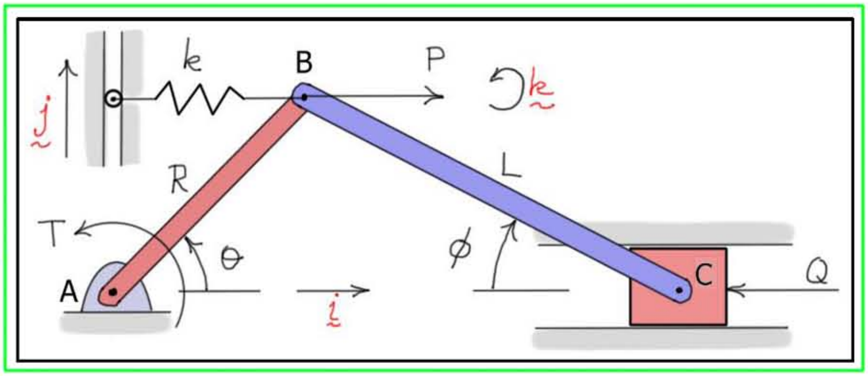
Example #18 – Intermediate Dynamics: Principle of Virtual Work

Active Forces/Torques:

- P, Q, T (neglecting weight forces)
- spring (unstretched length, ℓ_u)

Find:

- Force Q required to hold the system in equilibrium at some angle θ



Solution: (using θ as the generalized coordinate)

For equilibrium of this system, the ***principle of virtual work*** states

$$\boxed{F_\theta = (F_\theta)_P + (F_\theta)_Q + (F_\theta)_{spring} + (F_\theta)_T = 0}$$

Here,

$$(F_\theta)_P = (P\underset{\sim}{i})\cdot\left(\frac{\partial \underset{\sim}{v}_B}{\partial\dot{\theta}}\right) = (P\underset{\sim}{i})\cdot\frac{\partial}{\partial\dot{\theta}}\Big(R\dot{\theta}(-S_\theta\underset{\sim}{i}+C_\theta\underset{\sim}{j})\Big) = (P\underset{\sim}{i})\cdot\Big(R(-S_\theta\underset{\sim}{i}+C_\theta\underset{\sim}{j})\Big) = \boxed{-PRS_\theta}$$

$$(F_\theta)_{spring} = (-f_{sp}\underset{\sim}{i})\cdot\left(\frac{\partial \underset{\sim}{v}_B}{\partial\dot{\theta}}\right) = (-f_{sp}\underset{\sim}{i})\cdot\Big(R(-S_\theta\underset{\sim}{i}+C_\theta\underset{\sim}{j})\Big) = f_{sp}RS_\theta = \boxed{k(RC_\theta-\ell_u)RS_\theta}$$

$$(F_\theta)_Q = (-Q\underset{\sim}{i})\cdot\left(\frac{\partial \underset{\sim}{v}_C}{\partial\dot{\theta}}\right) = (-Q\underset{\sim}{i})\cdot\Big[-R\big(S_\theta + C_\theta S_\phi/C_\phi\big)\underset{\sim}{i}\Big] = \boxed{QR\big(S_\theta + C_\theta S_\phi/C_\phi\big)}$$

$$(F_\theta)_T = (T\underset{\sim}{k})\cdot\left(\frac{\partial \underset{\sim}{\omega}_{AB}}{\partial\dot{\theta}}\right) = T\underset{\sim}{k}\cdot\underset{\sim}{k} = \boxed{T}$$

Substituting into the principle of virtual work and solving for Q gives

$$F_\theta = 0 = -PRS_\theta + k(RC_\theta-\ell_u)RS_\theta + QR\big(S_\theta + C_\theta S_\phi/C_\phi\big) + T$$

$$\Rightarrow \quad \boxed{Q = \frac{PRS_\theta - k(RC_\theta-\ell_u)RS_\theta - T}{R\big(S_\theta + C_\theta S_\phi/C_\phi\big)}}$$

Notes:

- Pin forces at A, B, and C and normal force at C do not contribute to F_θ ("***non-active***")
- Forces and torques that contribute to F_θ are said to be "***active***"
- There is only ***one equation*** associated with the ***degree-of-freedom*** of the system.
- The ***choice*** of generalized coordinate is ours.
- The contribution of the spring could be calculated using ***potential energy***.

$$(F_\theta)_{spring} = -\frac{\partial V_{sp}}{\partial\theta} = -\frac{\partial}{\partial\theta}\left(\tfrac{1}{2}ke^2\right) = -\tfrac{1}{2}k\frac{\partial}{\partial\theta}\left(RC_\theta-\ell_u\right)^2 = \boxed{k(RC_\theta-\ell_u)RS_\theta}$$