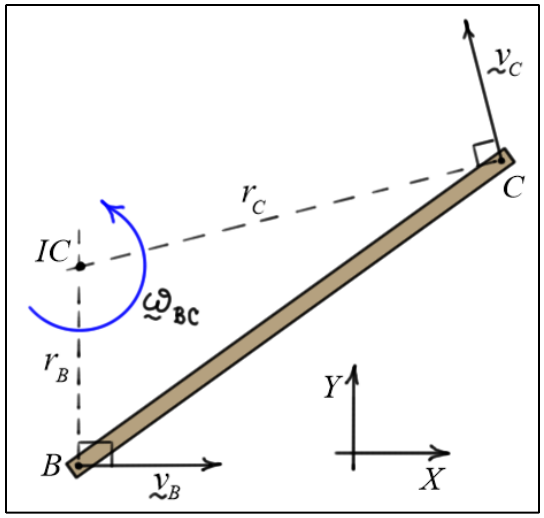
Elementary Engineering Mathematics
Application of Geometry/Trigonometry – Elementary Dynamics

The ***two-dimensional motion*** of a rigid body at any instant of time as it moves in the *XY* plane can be described as ***pure rotational motion*** about an ***instantaneous center*** (*IC*) of ***zero velocity***. The ***location*** of the *IC* relative to the body (at that instant) can be found by constructing lines ***perpendicular*** to the velocities of two points on the body. The ***intersection*** of these two lines (shown as dashed lines in the figure) is the location of the instantaneous center *IC*. Note from instant to instant, the *IC* ***changes location*** relative to the body.



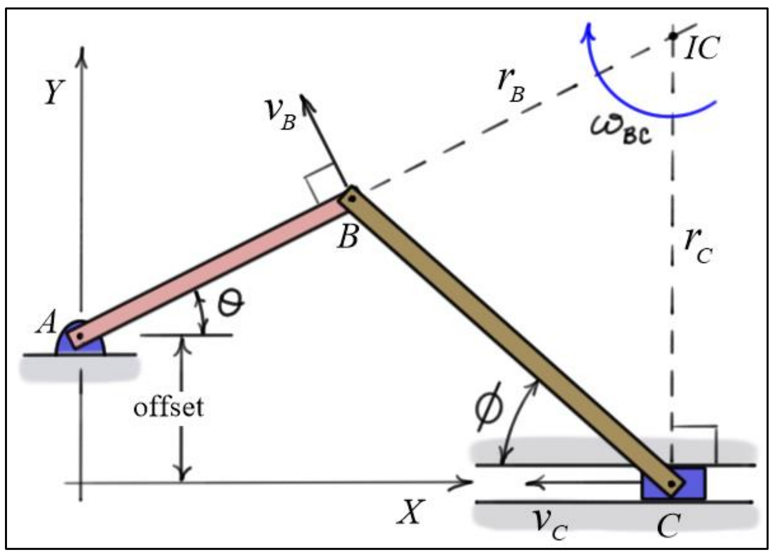
$$\omega_{BC} = \frac{v_B}{r_B} = \frac{v_C}{r_C} \text{ (radians/sec)}$$

The scalar ***angular velocity*** of the body ω_{BC} (i.e. how fast it is rotating about the *IC* in radians/second) is related to the scalar ***velocities*** of the two points as shown.

<u>Example:</u> Slider-crank mechanism

A slider-crank mechanism with an ***offset*** is shown in the diagram. Bar *AB* is the ***crank***, piston *C* is the ***slider***, and bar *BC* is the ***connecting rod***. In the position shown, as crank *AB* rotates ***counterclockwise***, the slider moves to the ***left***. The velocity of *B* is perpendicular to *AB*, and the velocity of *C* is to the left along the slot (*X*-axis).

The ***instantaneous center*** (*IC*) of connecting rod *BC* at ***this instant*** is found by constructing the dashed lines ***perpendicular*** to the ***velocities*** of points *B* and *C*. One of these lines is ***along*** crank *AB* and the other is ***perpendicular*** to the slot at *C*. The ***intersection*** point of these two lines is the ***instantaneous center***.

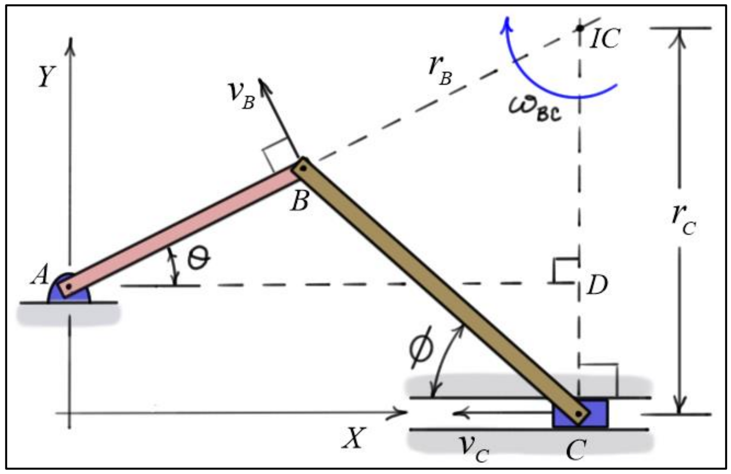


Problem:

Given: The ***coordinates*** of points A, B, and C (in inches) and the ***velocity*** of B:

A: (0, 3) B: (4, 5) C: (8,0)

$\boxed{v_B = 5 \text{ (in/s)}}$ in direction shown



Find: the ***location*** of IC and v_C the ***velocity*** of point C at this instant.

Solution #1: (using right triangles)

a) First, construct the dashed lines to the ***instantaneous center***. Then ***construct*** the ***right triangle*** $ADIC$, and ***calculate*** the angle of AB relative to AD.

$$\boxed{\theta = \tan^{-1}\left(\frac{y_B - y_A}{x_B - x_A}\right) = \tan^{-1}\left(\frac{5-3}{4-0}\right) = \tan^{-1}\left(\frac{2}{4}\right) \approx 26.565^{\text{o}}}$$

b) ***Calculate*** the distances r_B and r_C.

$$\boxed{\tan(\theta) = \frac{r_C - L_{CD}}{L_{AD}}} \quad \Rightarrow \boxed{r_C = L_{CD} + \left(L_{AD}\tan(\theta)\right) = 3 + \left(8 \times \tfrac{2}{4}\right) = 7 \text{ (in)}}$$

$$\boxed{\cos(\theta) = \frac{L_{AD}}{r_B + L_{AB}}} \quad \Rightarrow \boxed{\begin{aligned} r_B &= \left(\frac{L_{AD}}{\cos(\theta)}\right) - L_{AB} \approx \left(\frac{8}{\cos(26.565)}\right) - \sqrt{4^2 + 2^2} \\ &\approx 4.47214 \\ &\approx 4.47 \text{ (in)} \end{aligned}}$$

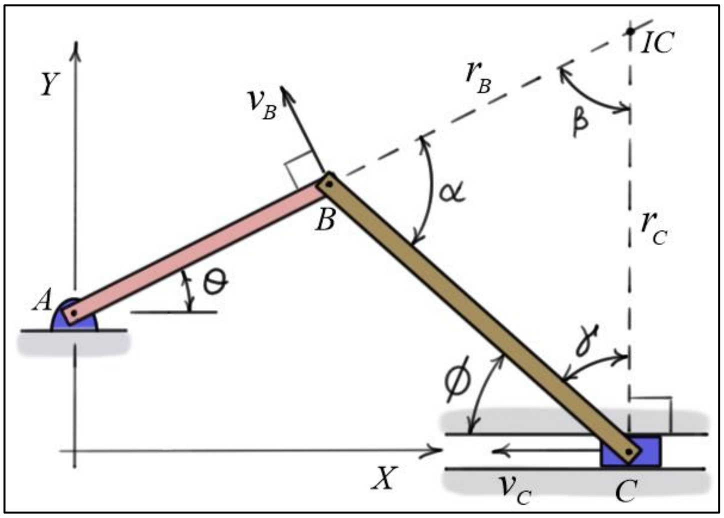
c) Find the ***angular velocity*** of BC and the ***velocity*** of piston C.

$$\boxed{\omega_{BC} = \frac{v_B}{r_B} \approx \frac{5 \text{ (in/s)}}{4.47214 \text{ (in)}} \approx 1.118 \approx 1.12 \text{ (rad/sec)}}$$ (angular motion is ***clockwise***)

$$\boxed{\frac{v_B}{r_B} = \frac{v_C}{r_C}} \quad \Rightarrow \boxed{v_C = \left(\frac{r_C}{r_B}\right) v_B \approx \left(\frac{7}{4.47214}\right) 5 \approx 7.82624 \approx 7.83 \text{ (in/s)}}$$

Note: When analyzing slider-crank mechanisms, we have the ***advantage*** of being able to use ***right triangles***; however, for more complex mechanisms (such as ***four-bar*** mechanisms), we will often need a more ***general approach*** using ***non-right triangles***.

Solution #2: (using non-right triangles)



a) First, calculate the angles of *AB* and *BC* relative to the *X*-axis.

$$\boxed{\theta = \tan^{-1}\left(\frac{y_B - y_A}{x_B - x_A}\right) = \tan^{-1}\left(2/4\right) \approx 26.565^{\circ}}$$

$$\boxed{\phi = \tan^{-1}\left(\frac{y_B - y_C}{x_C - x_B}\right) = \tan^{-1}\left(5/4\right) \approx 51.34^{\circ}}$$

b) Construct the dashed lines to the ***instantaneous center***. In the newly constructed triangle *BCIC*, define the ***unknown angles*** α, β, and γ.

c) ***Calculate*** the ***unknown angles*** using the concepts from geometry.

$\boxed{\alpha = \theta + \phi \approx 77.905^{\circ}}$ $\boxed{\beta = 90 - \theta \approx 63.435^{\circ}}$ $\boxed{\gamma = 90 - \phi \approx 38.66^{\circ}}$ … ***why?***

d) Now use the ***law of sines*** to find the distances r_B and r_C.

$$\boxed{\frac{\sin(\beta)}{b} = \frac{\sin(\gamma)}{r_B}} \Rightarrow \boxed{\frac{\sin(63.435)}{\sqrt{4^2+5^2}} = \frac{\sin(38.66)}{r_B}} \Rightarrow \boxed{r_B = \frac{\sin(38.66)}{\sin(63.435)}\sqrt{4^2+5^2}}$$

$$\Rightarrow \boxed{r_B \approx 4.47215 \approx 4.47 \text{ (inches)}}$$

$$\boxed{\frac{\sin(\beta)}{b} = \frac{\sin(\alpha)}{r_C}} \Rightarrow \boxed{\frac{\sin(63.435)}{\sqrt{4^2+5^2}} = \frac{\sin(77.905)}{r_C}} \Rightarrow \boxed{r_C = \frac{\sin(77.905)}{\sin(63.435)}\sqrt{4^2+5^2}}$$

$$\Rightarrow \boxed{r_C \approx 7.0 \text{ (inches)}}$$

e) Find the ***angular velocity*** of *BC* and the ***velocity*** of piston *C* as shown above.