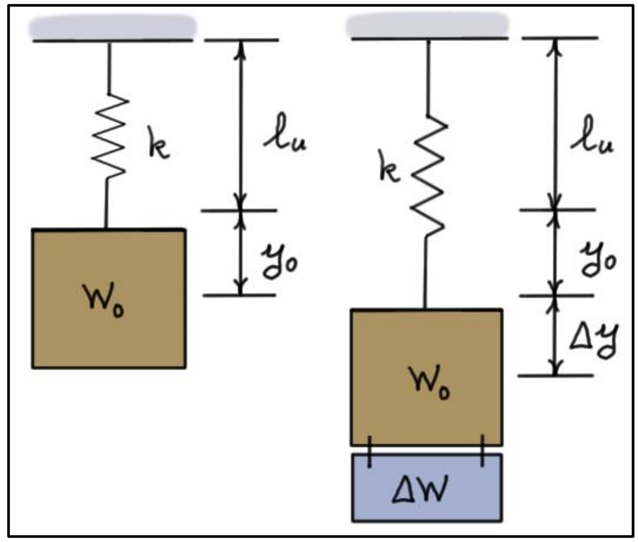
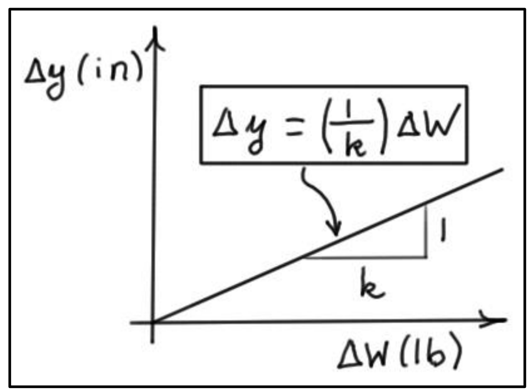
Elementary Engineering Mathematics
Application of Lines – Elementary Statics, Mechanics of Materials, and Dynamics

Example #1



Given: Consider a weight $W_0 = 17.3\,(\text{lb})$ which is supported by a ***linear spring*** of stiffness *k*. The length ℓ_u is the ***unstretched length*** of the spring, and y_0 is the required elongation of the spring to hold the weight W_0. As additional weights (ΔW) are added, the spring stretches more (Δy) to hold the additional weight. During this experiment, the additional displacement Δy can be related to the added weight ΔW using the equation of a line.



By adding known weights to the system and measuring the subsequent displacement changes, the following data were collected:

Weight, ΔW (lb)	Displacement, Δy (in)
10	1.21
20	2.45

Find:

a) estimate of the ***spring stiffness*** *k* (lb/ft), b) estimate of the ***initial displacement*** y_0 (in), and

c) an equation for the ***total displacement*** y as a function of ΔW.

Solution:

a) The slope of the line is $m = \dfrac{\Delta y}{\Delta W} = \dfrac{1}{k}$, so $\boxed{k = \dfrac{\Delta W}{\Delta y}}$

Weight, ΔW (lb)	Displacement, Δy (in)	Stiffness, k (lb/in)
10	1.21	8.2645
20	2.45	8.1633
	Average	8.214

An estimate of the spring stiffness is the average derived from the two measurements.

Units change: $\boxed{k \approx \left[\frac{8.214\,(\text{lb})}{\cancel{(\text{in})}}\right] \times \left[\frac{12\ \cancel{(\text{in})}}{(\text{ft})}\right] \approx 98.57\,(\text{lb/ft})}$

Here the symbol "≈" is used to indicate an approximate value.

b) The initial displacement y_0 can be found by noting that the initial displacement and weight are related by the same stiffness.

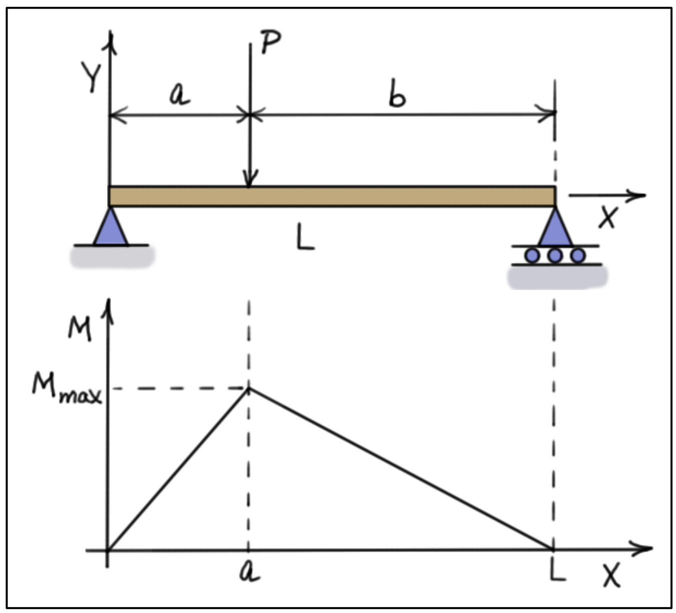
$$\boxed{y_0 = W_0 / k = 17.3\,(\text{lb}) / 8.214\,(\text{lb/in}) \approx 2.106\,(\text{in})}$$

c) The equation for the total displacement can be found using the ***slope-intercept*** form.

$$\boxed{y = 2.106 + (1/8.214)\Delta W = 2.106 + (0.1217)\,\Delta W}$$

Example #2

Given: Consider a ***long slender beam*** of length L with a ***concentrated load*** P acting at distance a from the left end. Due to this load, the beam experiences an ***internal bending moment*** M that varies linearly across the length of the beam as shown. The maximum bending moment $M_{\max}$ occurs at the load.



In an experiment, a load $P = 100$ (lb) is applied to a beam of length $L = 5$ (ft). The bending moments measured at two points on either side of P are given in the following table.

Location, x (ft)	Moment, M (ft-lb)	Location Relative to Load
2.067	64.3	left of load
4.378	42.87	right of load

Find:

a) the ***moment equations*** for $0 \le x \le a$ and $a \le x \le L$; b) the ***location*** of the load P; and c) the ***maximum moment*** experienced by the beam.

Solution:

a) For $0 \le x \le a$, the slope of the line is $\boxed{m = \dfrac{M\big|_{x=2.067} - M\big|_{x=0}}{2.067-0} = \dfrac{64.3-0}{2.067-0} \approx 31.11}$, so

$$\boxed{M(x) = m\,x = 31.11\,x} \qquad (1)$$

For $a \le x \le L$, the slope of the line is $\boxed{m = \dfrac{M\big|_{x=5} - M\big|_{x=4.378}}{5-4.378} = \dfrac{0-42.87}{5-4.378} \approx -68.92}$

Using the point-slope form of a line, we can write

$$(M - 42.87) = -68.92(x - 4.378) \quad \Rightarrow M = \big(42.87 + (68.92 \times 4.378)\big) - 68.92\,x$$

or

$$\boxed{M(x) = 344.6 - 68.92\,x} \qquad (2)$$

b) The load P is located at the point $x = a$ where moment equations (1) and (2) are equal, that is, at the ***intersection*** of the two lines. To find a, set

$$31.11x = 344.6 - 68.92\,x \quad \Rightarrow \quad (31.11 + 68.92)\,x\big|_{x=a} = 344.6$$

$$\Rightarrow \quad \boxed{a = \frac{344.6}{31.11 + 68.92} \approx 3.445 \text{ (ft)}}$$

c) The ***maximum moment*** experienced by the beam can be calculated by substituting the value of a into equations (1) or (2).

$\boxed{M_{\max} = M(a) = 31.11 \times 3.445 = 107.2 \text{ (ft-lb)}}$ or $\boxed{M_{\max} = 344.6 - (68.92 \times 3.445) = 107.2 \text{ (ft-lb)}}$