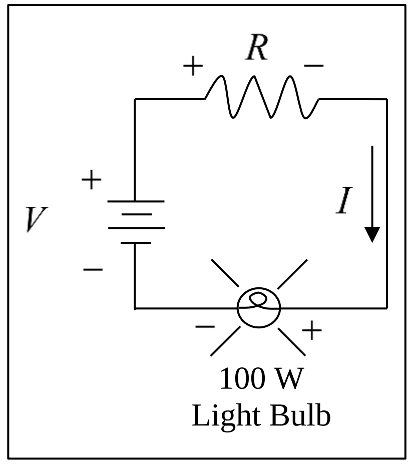
Elementary Engineering Mathematics
Application of Quadratic Equations in Electric Circuits



Example #1

Given: A 100-watt light bulb is connected in ***series*** with a resistor $R = 10\ \text{(ohms)}$. The applied voltage is $V = 120\ \text{(volts)}$. The ***power*** used by the light bulb is calculated as $P_L = V_L\, I$. The units of power are "volt-amps" or "watts."

Find: The current I in amperes (amps).

Solution:

To find the current, we first apply ***Kirchhoff's voltage law*** to the circuit.

$$\boxed{V = 120 = V_R + V_L} \qquad (1)$$

Then, represent the voltage drops across the resistor and the light bulb in terms of the current I.

Ohm's law: $\boxed{V_R = R\, I = 10\, I}$ (2)

Power equation: $P_L = V_L\, I \quad \Rightarrow \quad \boxed{V_L = P_L / I = 100/I}$ (3)

Substituting from equations (2) and (3) into equation (1), gives

$$\boxed{120 = (10\, I) + (100/I)} \quad \Rightarrow 120\, I = 10\, I^2 + 100 \quad \Rightarrow \boxed{10\, I^2 - 120\, I + 100 = 0} \qquad (4)$$

Quadratic formula

Using the quadratic formula, the roots of equation (4) are calculated as follows

$$\boxed{I_{1,2} = \frac{120 \pm \sqrt{120^2 - (4 \times 10 \times 100)}}{2 \times 10} \approx \frac{120 \pm 101.98}{20} \approx \begin{cases} 11.1\ \text{(amps)} \\ 0.901\ \text{(amps)} \end{cases}} \qquad (5)$$

Completing the Square

To complete the square, we first divide equation (4) by 10: $\boxed{I^2 - 12\, I + 10 = 0}$

$$I^2 - 12\, I + \left(\tfrac{12}{2}\right)^2 = -10 + \left(\tfrac{12}{2}\right)^2 \quad \Rightarrow \quad I^2 - 12\, I + 6^2 = -10 + 6^2$$

$$\Rightarrow (I - 6)^2 = 26 \quad \Rightarrow \quad I - 6 = \pm\sqrt{26} \approx \pm 5.09902$$

$$\Rightarrow \boxed{I \approx 6 \pm 5.09902 \approx \begin{cases} 11.1\ \text{(amps)} \\ 0.901\ \text{(amps)} \end{cases}}$$

Factoring

As the roots are not integers, it is not easy factor the quadratic equation directly. However, given the above results, we recognize that

$$\boxed{I^2 - 12\,I + 10 \approx (I - 11.1)(I - 0.901)}$$

Back to our circuit

Question: How is it that our circuit can have two different currents?

Answer: Actually, it does not. The power of the light bulb is ***rated*** at a ***specific voltage***, so only one of the currents is correct for a given light bulb.

Let's calculate the voltage and resistance of the light bulb at each of these currents.

a) $I \approx 0.901 \text{ (amps)} \Rightarrow \boxed{V_L = P_L / I \approx 100/0.901 \approx 111 \text{ (volts)}}$

$$\Rightarrow \boxed{R_L = V_L / I \approx 111/0.901 \approx 123 \text{(ohms)}}$$

b) $I \approx 11.1 \text{ (amps)} \Rightarrow \boxed{V_L = P_L / I \approx 100/11.1 \approx 9.01 \text{ (volts)}}$

$$\Rightarrow \boxed{R_L = V_L / I \approx 9.01/11.1 \approx 0.812 \text{(ohms)}}$$

Summary @ $V = 120$ (volts) and $R = 10$ (ohms)

I (amps)	$V_R = R\,I$ (volts)	$P_R = V_R\,I$ (watts)	$V_L = V - V_R$ (volts)	$R_L = V_L / I$ (ohms)	$P_L = V_L\,I$ (watts)
0.901	9.01	8.12	111	123	100
11.1	111	1232	9	0.812	100

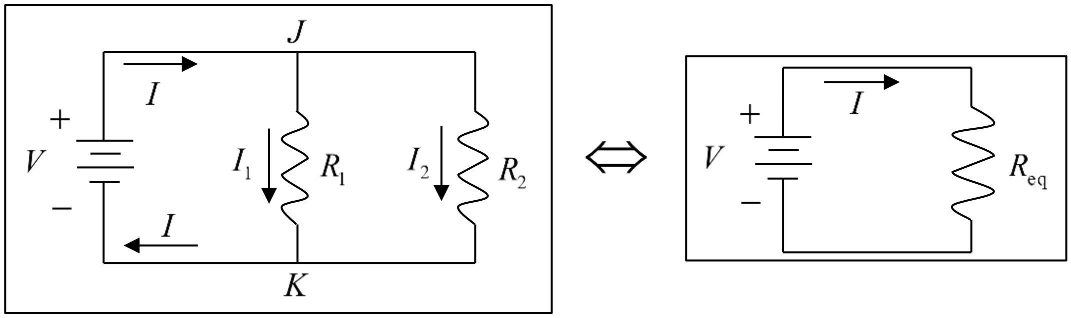
Note: The resistance of an average 100-watt light bulb at 120 volts is about 140 ohms.

Example #2

Given: The electric circuit shown below has two resistors connected in ***parallel***. At node J the current I splits into two parts, I_1 and I_2, and at node K the currents recombine to form I. The splitting and combining of currents at nodes obey ***Kirchhoff's Current Law***.

Kirchhoff's Current Law:

The ***sum*** of the currents ***flowing into*** a node ***equals*** to the sum of the currents ***flowing away*** from a node. (In this case, $I = I_1 + I_2$.)



Equivalent Electrical Circuits

Using Kirchhoff's law, it can be shown that the two parallel resistors (R_1 and R_2) act as a single ***equivalent*** resistor R_{eq}.

$$\boxed{R_{eq} = \frac{R_1 R_2}{R_1 + R_2}} \tag{6}$$

Finally, it is known that $\boxed{R_{eq} = 150 \text{ (ohms)}}$ and $\boxed{R_2 = R_1 + 200}$.

Find: The resistances R_1 and R_2.

Solution:

Substituting the known information into equation (6) gives

$$R_{eq} = 150 = \frac{R_1 R_2}{R_1 + R_2} = \frac{R_1(R_1 + 200)}{R_1 + (R_1 + 200)} = \frac{R_1^2 + 200R_1}{2R_1 + 200}$$

Clearing fractions and combining terms gives

$$150(2R_1 + 200) = R_1^2 + 200R_1 \quad \Rightarrow \boxed{R_1^2 + \underbrace{(200 - 300)}_{-100} R_1 - \underbrace{(150 \times 200)}_{30000} = 0}$$

Using the quadratic formula, we find

$$R_1 = \frac{100 \pm \sqrt{100^2 - 4(-30000)}}{2} \approx \frac{100 \pm 360.55}{2} \approx \begin{cases} \boxed{230.28} \\ -130.28 \end{cases}$$

As the value of resistance cannot be negative, we find $\boxed{R_1 \approx 230 \text{ (ohms)}}$ and $\boxed{R_2 \approx 430 \text{ (ohms)}}$.