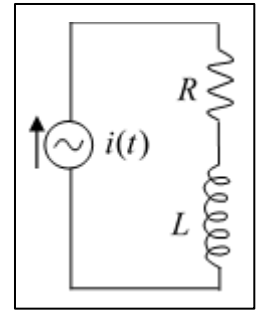


Elementary Engineering Mathematics

Application of Sine and Cosine Functions in Electrical Engineering

Given: A current source $i(t) = 2 \sin(120\pi t)$ is applied to an RL series circuit with $R = 100$ (ohms) and $L = 100$ (mh). The voltage drop across the resistor is $V_R = Z_R I$, the voltage drop across the inductor is $V_L = Z_L I$, and the total voltage drop across both elements is $V = V_R + V_L$.



Find: (a) $v(t)$ the voltage as a function of time. (b) $v(t)$ as a single, phase-shifted *sine wave*.
(c) $v(t)$ as a single, phase-shifted *cosine wave*.

Solution:

a) $V_R = Z_R I = (100 \angle 0^\circ)(2 \angle -90^\circ) = 200 \angle -90^\circ$

or $v_R(t) = 200 \cos(120\pi t - 90) = 200 \sin(120\pi t)$

$$V_L = Z_L I = ((120\pi(0.1)) \angle 90^\circ)(2 \angle -90^\circ) \approx 75.4 \angle 0^\circ \quad \text{or} \quad v_L(t) \approx 75.4 \cos(120\pi t)$$

So,

$$v(t) \approx 200 \sin(120\pi t) + 75.4 \cos(120\pi t)$$

b) $v(t) = M \sin(120\pi t + \phi)$

$$M = \sqrt{200^2 + 75.4^2} \approx 214 \text{ (volts)} \quad \phi \approx \tan^{-1}(75.4/200) \approx 20.66^\circ \approx 0.3605 \text{ (rad)}$$

c) $v(t) = M \cos(120\pi t + \phi)$

$$M = \sqrt{200^2 + 75.4^2} \approx 214 \text{ (volts)} \quad \phi \approx \tan^{-1}(-200/75.4) \approx -69.34^\circ \approx -1.21 \text{ (rad)}$$

Alternate Solution:

$$V_R = Z_R I = 200 \angle -90^\circ = -j200 \quad \text{and} \quad V_L = Z_L I \approx 75.4 \angle 0^\circ \approx 75.4$$

So,

$$V = 75.4 - j200 = (\sqrt{75.4^2 + 200^2}) \angle \tan^{-1}(-200/75.4) \approx 214 \angle (-1.21) \text{ (volts)}$$

Or,

$$\begin{aligned} v(t) &= 214 \cos(120\pi t - 1.21) = 214 \sin(120\pi t - 1.21 + \pi/2) \\ &= 214 \sin(120\pi t + 0.3605) \text{ (volts)} \end{aligned} \quad \text{(See plot below)}$$

