An Introduction to
Three-Dimensional, Rigid Body Dynamics

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Volume III – Introduction to Multibody Kinematics

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Unit 1 – Background Material

This unit introduces the use of ***matrix methods*** to perform the basic mathematical operations required for the development of the equations of motion for multibody systems. This allows the equations to be expressed in a more ***compact form*** that is easily converted into ***computer algorithms***. These algorithms can be used to ***reliably*** generate the equations of motion of complex systems rather than trying to generate them by hand. It should be stressed that a ***thorough understanding*** of the equations in this form depends heavily on the ***analyst's grasp*** of the ***vector-based methods*** presented in Volumes I and II.

This unit also reviews the concept of ***generalized coordinates***, ***generalized speeds***, ***state vectors***, and ***body-connection*** and ***ending-body*** arrays.

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Unit 2 – Angular Velocity and Partial Angular Velocity

This unit focuses on the ***matrix-based*** calculation of ***vector*** components of ***angular velocity*** and ***partial angular velocity matrices***. The calculations are performed using ***fixed frame*** and ***body frame*** components and are based on ***absolute*** and ***relative coordinates***. Both ***orientation angle derivatives*** and ***angular velocity*** components are used as ***generalized speeds***. Algorithms are developed for the efficient calculation of these quantities for multibody systems.

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Unit 3 – Angular Acceleration

This unit focuses on the ***matrix-based*** calculation of ***vector*** components of ***angular acceleration***. The calculations are performed using ***fixed frame*** and ***body frame*** components and are based on ***absolute*** and ***relative coordinates***. Both ***orientation angle derivatives*** and ***angular velocity components*** are used as ***generalized speeds***. As part of the calculations, a procedure for calculating the ***time derivatives*** of ***relative transformation matrices*** (between moving bodies) is also included.

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35	4	10

Unit 4 – Velocity and Partial Velocity

This unit focuses on the ***matrix-based*** calculation of ***vector*** components of ***velocity*** and ***partial velocity matrices***. The calculations are based on ***absolute*** and ***relative coordinates***. Both ***orientation angle derivatives*** and ***angular velocity components*** can be used as ***generalized speeds***. Algorithms are developed for the efficient calculation of these quantities for multibody systems.

Explicit results are generated for some ***examples*** with the ***purpose*** of being ***clear*** about ***how*** the calculations are done. However, keep in mind that the ***goal*** of developing such procedures is to ***implement*** them into ***computer algorithms***.

Results for calculating the ***derivatives*** of ***transformation matrices*** presented in Units 1 and 3 of this volume are ***repeated*** in this unit for convenience of the reader.

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Unit 5 – Acceleration

This unit focuses on ***matrix-based*** calculations of ***fixed frame components*** of ***acceleration***. The accelerations are expressed in terms of partial velocity matrices and their time derivatives. Both ***base frame*** and ***body frame*** components of ***relative angular velocity vectors*** are used in the calculations. Both ***absolute*** and ***relative coordinates*** are discussed. Both ***orientation angle derivatives*** and ***angular velocity components*** can be used as ***generalized speeds***.

Explicit results are generated for some ***examples*** with the ***purpose*** of being ***clear*** about ***how*** the calculations are done. However, keep in mind that the ***goal*** of developing such procedures is to ***implement*** them into ***computer algorithms***.

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References:

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