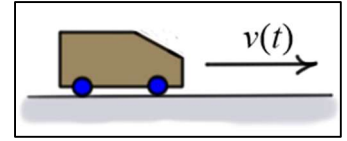**Elementary Engineering Mathematics**
Application of Lines in Elementary Dynamics

Example #1

Given: Consider a car moving with **velocity** $v(t)$. For a **constant braking force**, the velocity of the car satisfies the equation:

$$\boxed{v(t) = v_0 + a_0 t} \tag{1}$$

Here, v_0 is the **velocity** of the car at the time the **brakes** are **applied**, a_0 is the **constant acceleration** of the car until it stops, and t is the time. During a **test** of the car's braking system, the following data were measured:

Time, t (s)	Velocity, $v(t)$ (ft/s)	Velocity, $v(t)$ (mi/hr)
2.9	74.5	50.8
7.2	30.2	20.6

Find: a) a_0 the **constant acceleration** of the car; b) v_0 the **initial velocity** of the car; and

    c) t^* the **time required** for the car to **stop**. Assume a **constant breaking force** is applied.

Solution:

Equation (1) is in the **slope-intercept form** of the equation for a line: $\boxed{y = m x + b}$. Here, the **slope** of the line is $m = a_0$ and the y-intercept is $b = v_0$.

a) The slope a_0 can be **estimated** using the **recorded data**.

$$\boxed{a_0 = m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{30.2 - 74.5}{7.2 - 2.9} \approx -10.30 \,(\text{ft/s}^2)}$$

Note: The symbol "$\approx$" is used here to indicate an **approximate** value.

So, we now have

$$\boxed{v(t) = -10.30t + v_0}$$

b) The *y*-intercept v_0 can now be found by using the **slope** and **either** of the **two data pairs**.

$$v(t)\big|_{t=2.9} = 74.5 \approx -(10.30 \times 2.9) + v_0 \quad \Rightarrow \quad \boxed{v_0 \approx 74.5 + (10.30 \times 2.9) \approx 104.4 \,(\text{ft/s})}$$

or

$$v(t)\big|_{t=7.2} = 30.2 \approx -(10.30 \times 7.2) + v_0 \quad \Rightarrow \quad \boxed{v_0 \approx 30.2 + (10.30 \times 7.2) \approx 104.4 \,(\text{ft/s})}$$
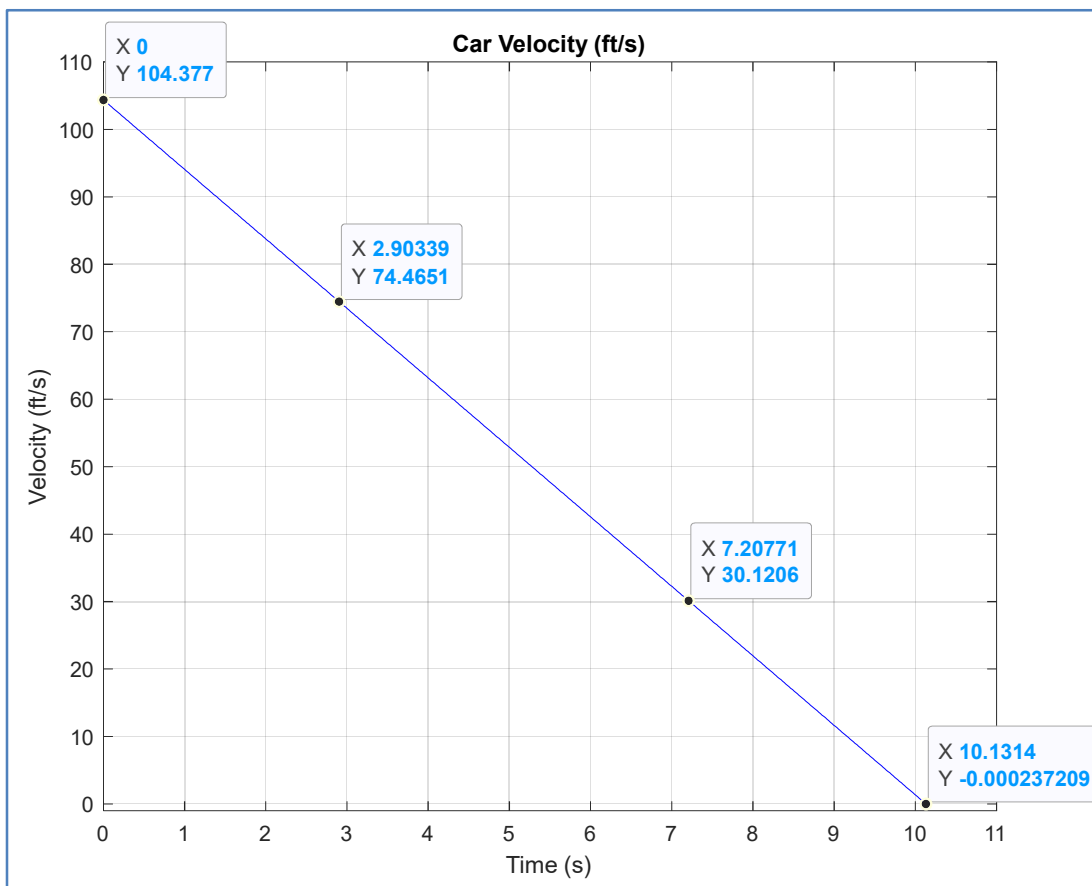
We now have the **completed velocity equation**: $\boxed{v(t) = -10.30t + 104.4}$ (ft/s) (2)

c) Using equation (2), we can find the time t^* required for the car to **stop**.

$$v(t^*) = 0 = 104.4 - (10.30t^*) \quad \Rightarrow \quad \boxed{t^* \approx 104.4 / 10.30 \approx 10.14 \,(s)}$$
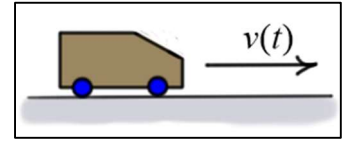
Note:

The **stopping time** t^* and the **initial velocity** v_0 are the **x**- and **y-intercepts** of the line.

Example #2

Given: Again, consider a car moving with **velocity** $v(t)$. As before, for a **constant braking force**, the velocity of the car satisfies the equation:

$$\boxed{v(t) = v_0 + a_0 t} \tag{3}$$

During a **second test** of the car's braking system, the following data were measured:

Time, t (s)	Velocity, $v(t)$ (ft/s)	Acceleration, a (ft/s^2)
4.3	59.7	-10.5

Find: a) v_0 the **initial velocity** of the car; and b) t^* the time required for the car to **stop**.
Assume a **constant breaking force** is applied.

Solution:

a) To find the **initial velocity** v_0, we can use the **point-slope form** of the equation for a line.

$$\frac{y - y_1}{x - x_1} = m \quad \Rightarrow \quad \frac{v(t) - 59.7}{t - 4.3} = -10.5 \quad \Rightarrow \quad v(t) - 59.7 = -10.5(t - 4.3)$$
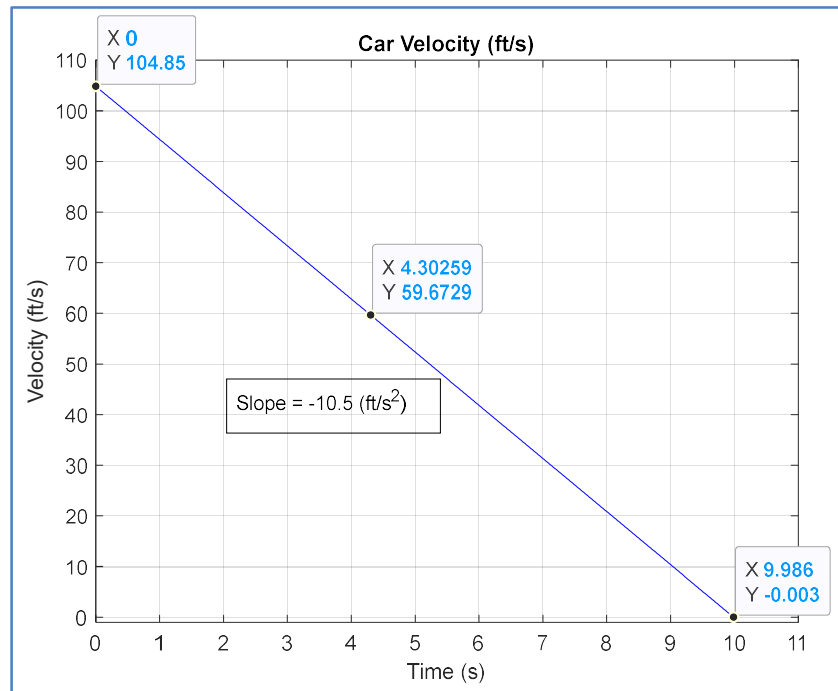
or

$$v(t) = \left(59.7 + (10.5 \times 4.3)\right) - 10.5t \quad \Rightarrow \quad \boxed{v(t) = 104.85 - 10.5t} \text{ (ft/s)} \tag{4}$$

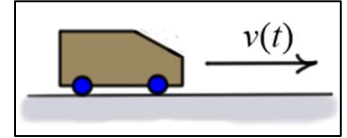Comparing equations (3) and (4) yields: $\boxed{v_0 = 104.85 \text{ (ft/s)} \approx 71.5 \text{ (mi/hr)}}$

b) Equation (4) can also be used to find the time t^*

$$v(t^*) = 0 = 104.85 - (10.5 \ t^*) \quad \Rightarrow \quad \boxed{t^* = 104.85 / 10.5 \approx 9.99 \text{ (s)}}$$

Car Velocity (ft/s)

X 0
Y 104.85

X 4.30259
Y 59.6729

Slope = -10.5 (ft/s^2)

X 9.986
Y -0.003

Velocity (ft/s)

Time (s)

Example #3

Given: Now, consider a car that **starts** at **rest**, **accelerates** at a **constant** **rate** of $a_0 = 14.8$ (ft/s^2) for 6 seconds, and then **decelerates** at a **constant** **rate** $a_1 = -10.5$ (ft/s^2) until it **stops**. Since the car **starts** from **rest**, during the **constant acceleration phase**, the velocity of the car satisfies the equation

$$\boxed{v(t) = a_0\,t = 14.8\,t} \tag{5}$$

During the **constant deceleration phase**, the velocity of the car satisfies the equation

$$\boxed{\frac{v(t) - v(t)\big|_{t=6}}{t-6} = a_1 = -10.5} \qquad \text{(Point-slope form)} \tag{6}$$

Find: a) the equation for $v(t)$ that applies during the **deceleration** phase, and b) t^* the time when the car stops.

Solution:

a) Using equation (5), we find the velocity of the car at $t = 6$ (sec) to be

$$\boxed{v(t)\big|_{t=6} = a_0\,t\big|_{t=6} = (14.8)(6) = 88.8 \text{ (ft/s)}}$$

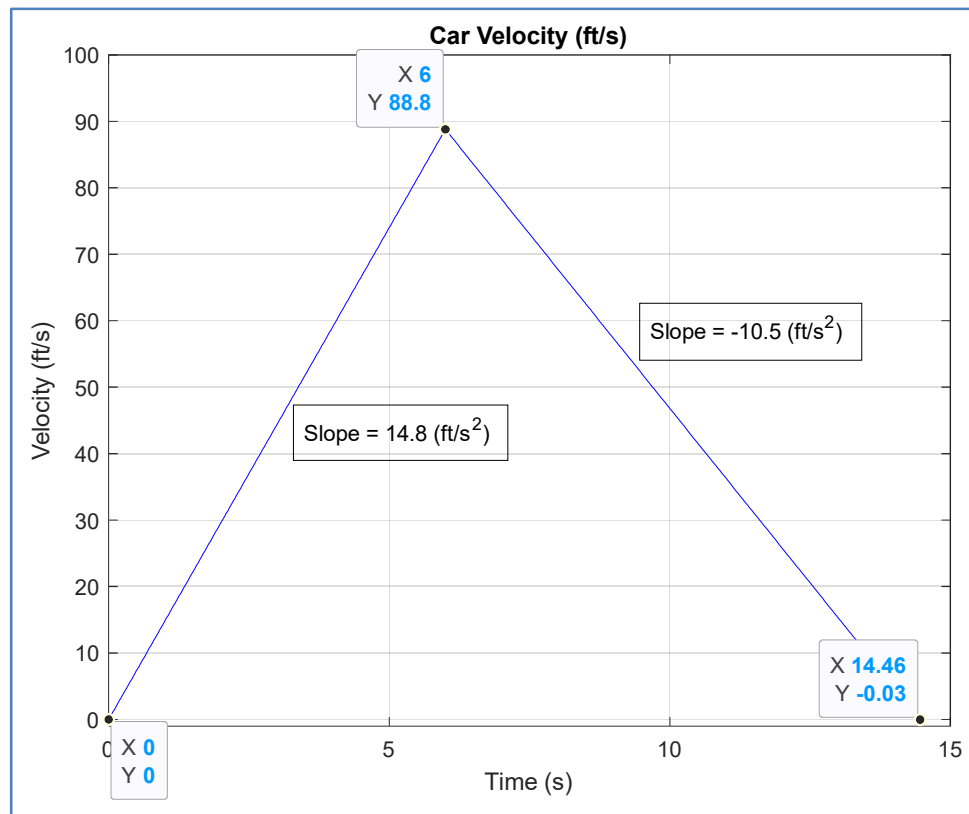Substituting into the point-slope form in equation (6) and reorganizing terms gives

$$\frac{v(t) - v(t)\big|_{t=6}}{t-6} = \frac{v(t) - 88.8}{t-6} = -10.5$$

$$\Rightarrow \quad v(t) - 88.8 = -10.5(t-6)$$

$$\Rightarrow \quad v(t) = \left(88.8 + (6 \times 10.5)\right) - 10.5t$$

or

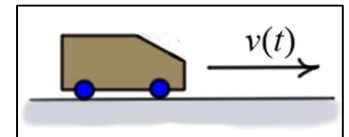$$\boxed{v(t) = 151.8 - 10.5t} \text{ (ft/s)}$$

b) To find the time t^* when the car stops, set

$$v(t^*) = 0 = 151.8 - (10.5t^*) \quad \Rightarrow \quad \boxed{t^* = 151.8 / 10.5 \approx 14.46 \text{ (s)}}$$

Car Velocity (ft/s)

Example #4

Example #4

Given: Now, consider a car that **starts** at **rest**, **accelerates** at a **constant rate** of a_1(m/s^2) for t_1 seconds, and then **decelerates** at a **constant rate** a_2(m/s^2) until it **stops** at time t_2. Since the car **starts** from **rest**, during the **constant acceleration phase**, the **velocity** of the car satisfies the equation

$$\boxed{v(t) = a_1 t} \qquad \left(a_1 > 0\right) \tag{7}$$

During the **constant deceleration phase**, the velocity of the car satisfies the equation

$$\boxed{\frac{v(t) - v(t)\big|_{t=t_1}}{t - t_1} = -a_2} \qquad \left(a_2 > 0\right) \qquad \text{(Point-slope form)} \tag{8}$$

Find: a) time t_2 in terms of time t_1 and the acceleration and deceleration rates a_1 and a_2, and b) $\left(t_2 - t_1\right)/t_1$ the ratio of the time durations of deceleration and acceleration.

<u>Solution:</u>

a) Using the **point-slope** form in equation (8), and **substituting** for $v(t_1)$ using equation (7) gives

$$\frac{v(t) - v(t)|_{t=t_1}}{t - t_1} = -a_2$$

$$\Rightarrow \quad v(t) - a_1 t_1 = -a_2(t - t_1)$$

$$\Rightarrow \quad \boxed{v(t) = (a_1 + a_2)\, t_1 - a_2 t}\ \text{(m/s)}$$

Now, using the fact that $v(t_2) = 0$, we can solve for the time t_2 as follows

$$v(t_2) = 0 = (a_1 + a_2)t_1 - a_2 t_2 \quad \Rightarrow \quad \boxed{t_2 = \left[\frac{a_1 + a_2}{a_2}\right] t_1}\ \text{(s)} \qquad\qquad (9)$$

b) Using equation (9), we can solve for the **ratio** of the **time durations** of the **deceleration** and **acceleration** phases.

$$t_2 - t_1 = \left[\frac{a_1 + a_2}{a_2}\right] t_1 - t_1 = \left[\frac{a_1 + a_2}{a_2} - 1\right] t_1 = \left[\frac{a_1 + a_2}{a_2} - \frac{a_2}{a_2}\right] t_1 = \left[\frac{a_1 + \cancel{a_2} - \cancel{a_2}}{a_2}\right] t_1 = \left[\frac{a_1}{a_2}\right] t_1$$

or

$$\boxed{\frac{t_2 - t_1}{t_1} = \frac{a_1}{a_2}}$$