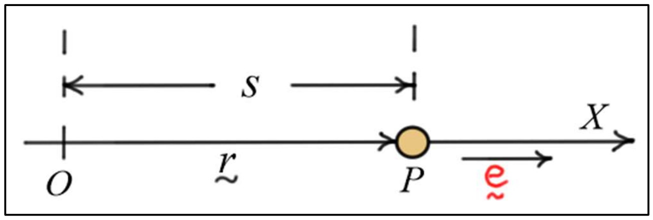
Elementary Dynamics
Rectilinear (Straight Line) Motion

General Concepts:

Position, Velocity, and Acceleration

A particle P has ***rectilinear*** motion when it moves in a straight line. As shown in the figure, define the direction of motion as the X-axis along which we define a ***unit vector***, $\underset{\sim}{e}$.



The ***position vector*** of P may then be written as $\underset{\sim}{r} = s\,\underset{\sim}{e}$, where s is the distance from some fixed point on the axis (in this case, O) to P. The ***velocity*** of P is defined as the derivative of the position vector. Using the ***product rule***, we can write

$$\underset{\sim}{v} = \frac{d\underset{\sim}{r}}{dt} = \frac{d}{dt}\left(s\underset{\sim}{e}\right) = \left(\frac{ds}{dt}\,\underset{\sim}{e}\right) + \left(s\,\frac{d\underset{\sim}{e}}{dt}\right) = \frac{ds}{dt}\underset{\sim}{e} = \dot{s}\underset{\sim}{e} = v\underset{\sim}{e}$$

Similarly, the ***acceleration*** of P is defined as the derivative of the velocity. So, the acceleration may be written as

$$\underset{\sim}{a} = \frac{d^2 s}{dt^2}\underset{\sim}{e} = \ddot{s}\,\underset{\sim}{e} = \dot{v}\,\underset{\sim}{e} = a\,\underset{\sim}{e}$$

Average Velocity, Average Speed, and Average Acceleration

If Δs and s_T represent the "displacement" and "total distance" traveled by P over the interval of time Δt, then the average velocity and average speed of P over the interval Δt are defined to be

Average Velocity: $\underset{\sim}{v}_{avg} = \dfrac{\Delta \underset{\sim}{r}}{\Delta t}$, Average Speed: $v_{avg} = \dfrac{s_T}{\Delta t}$.

The average acceleration over the time interval Δt is defined to be: $\underset{\sim}{a}_{avg} = \dfrac{\Delta \underset{\sim}{v}}{\Delta t}$.

Problem Solving:

The following list provides a guide to solving rectilinear motion problems. Given a position, velocity, or acceleration function, the list shows what you can calculate.

1. Given: $s = s(t)$ Calculate: $v(t) = \dfrac{ds}{dt}$ and $a(t) = \dfrac{dv}{dt}$

2. Given: $v = v(t)$ Calculate: $\int_{s_0}^{s} ds = s(t) - s(0) = \int_{t_0}^{t} v(t)dt$ and $a(t) = \dfrac{dv}{dt}$

3. Given: $a = a(t)$ Calculate: $\dfrac{dv}{dt} = a(t) \begin{cases} \int_{v_0}^{v} dv = v(t) - v(0) = \int_{t_0}^{t} a(t)dt \\ \int_{s_0}^{s} ds = s(t) - s(0) = \int_{t_0}^{t} v(t)dt \end{cases}$

4. Given: $a = a(s)$ Calculate: $v\dfrac{dv}{ds} = a(s) \quad \Rightarrow \begin{cases} \int_{v_0}^{v} v dv = \frac{1}{2}\left(v^2 - v_0^2\right) = \int_{s_0}^{s} a(s)ds \\ \dfrac{ds}{dt} = v(s) \quad \Rightarrow \quad \int_{s_0}^{s} \dfrac{ds}{v(s)} = \int_{t_0}^{t} dt = t - t_0 \end{cases}$

5. Given: $a = a(v)$ Calculate: $v\dfrac{dv}{ds} = a(v) \Rightarrow \int_{v_0}^{v} \dfrac{v dv}{a(v)} = \int_{s_0}^{s} ds = s(v) - s(0)$

6. Given: $a = a(v)$ Calculate: $\dfrac{dv}{dt} = a(v) \Rightarrow \begin{cases} \int_{v_0}^{v} \dfrac{dv}{a(v)} = \int_{t_0}^{t} dt = t - t_0 \\ \int_{s_0}^{s} ds = s(t) - s(0) = \int_{t_0}^{t} v(t)dt \end{cases}$

7. Given: $a = a_0 =$ constant

Calculate: $\dfrac{dv}{dt} = a_0 \Rightarrow \begin{cases} v(t) = v_0 + a_0(t - t_0) \\ s(t) = s_0 + v_0(t - t_0) + \frac{1}{2}a_0(t - t_0)^2 \end{cases}$

Calculate: $v\dfrac{dv}{ds} = a_0 \quad \Rightarrow \quad v^2(t) = v_0^2 + 2a_0(s - s_0)$