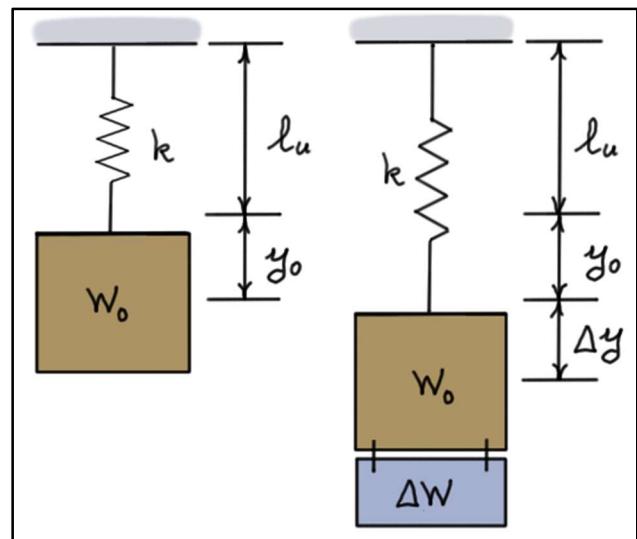


# Elementary Engineering Mathematics

## Application of Lines – Elementary Statics, Mechanics of Materials, and Dynamics

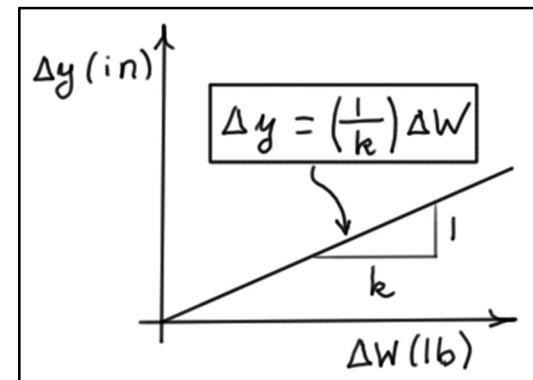
### Example #1

Given: Consider a weight  $W_0 = 17.3$  (lb) which is supported by a *linear spring* of stiffness  $k$ . The length  $\ell_u$  is the *unstretched length* of the spring, and  $y_0$  is the required elongation of the spring to hold the weight  $W_0$ . As additional weights ( $\Delta W$ ) are added, the spring stretches more ( $\Delta y$ ) to hold the additional weight. During this experiment, the additional displacement  $\Delta y$  can be related to the added weight  $\Delta W$  using the equation of a line.



By adding known weights to the system and measuring the subsequent displacement changes, the following data were collected:

| Weight, $\Delta W$ (lb) | Displacement, $\Delta y$ (in) |
|-------------------------|-------------------------------|
| 10                      | 1.21                          |
| 20                      | 2.45                          |



Find:

- estimate of the *spring stiffness*  $k$  (lb/ft),
- estimate of the *initial displacement*  $y_0$  (in), and
- an equation for the *total displacement*  $y$  as a function of  $\Delta W$ .

Solution:

a) The slope of the line is  $m = \frac{\Delta y}{\Delta W} = \frac{1}{k}$ , so  $k = \frac{\Delta W}{\Delta y}$

| Weight, $\Delta W$ (lb.) | Displacement, $\Delta y$ (in) | Stiffness, $k$ (lb/in) |
|--------------------------|-------------------------------|------------------------|
| 10                       | 1.21                          | 8.2645                 |
| 20                       | 2.45                          | 8.1633                 |
|                          | Average                       | 8.214                  |

An estimate of the spring stiffness is the average derived from the two measurements.

Units change: 
$$k \approx \left[ \frac{8.214 \text{ (lb)}}{\text{in}} \right] \times \left[ \frac{12 \text{ (in)}}{\text{ft}} \right] \approx 98.57 \text{ (lb/ft)}$$

Here the symbol “ $\approx$ ” is used to indicate an approximate value.

b) The initial displacement  $y_0$  can be found by noting that the initial displacement and weight are related by the same stiffness.

$$y_0 = W_0 / k = 17.3 \text{ (lb)} / 8.214 \text{ (lb/in)} \approx 2.106 \text{ (in)}$$

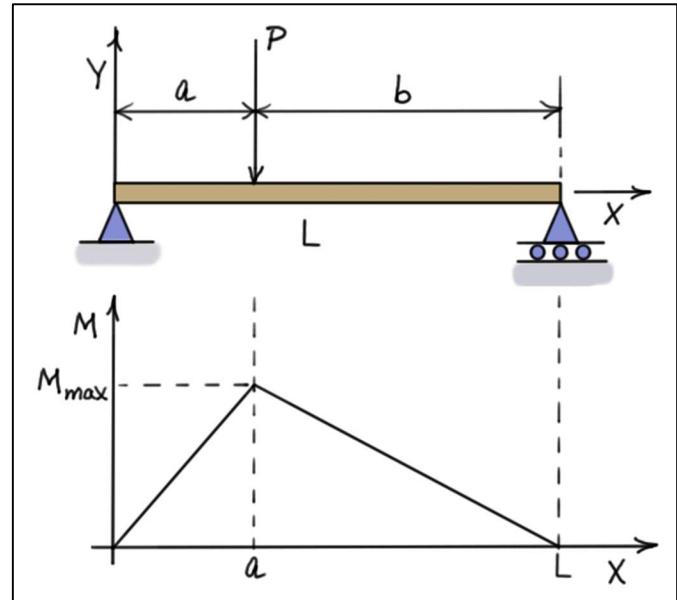
c) The equation for the total displacement can be found using the *slope-intercept* form.

$$y = 2.106 + (1/8.214)\Delta W = 2.106 + (0.1217)\Delta W$$

### Example #2

Given: Consider a **long slender beam** of length  $L$  with a **concentrated load**  $P$  acting at distance  $a$  from the left end. Due to this load, the beam experiences an **internal bending moment**  $M$  that varies linearly across the length of the beam as shown. The maximum bending moment  $M_{\max}$  occurs at the load.

In an experiment, a load  $P = 100 \text{ (lb)}$  is applied to a beam of length  $L = 5 \text{ (ft)}$ . The bending moments measured at two points on either side of  $P$  are given in the following table.



| Location, $x$ (ft) | Moment, $M$ (ft-lb) | Location Relative to Load |
|--------------------|---------------------|---------------------------|
| 2.067              | 64.3                | left of load              |
| 4.378              | 42.87               | right of load             |

Find:

a) the **moment equations** for  $0 \leq x \leq a$  and  $a \leq x \leq L$ ; b) the **location** of the load  $P$ ; and c) the **maximum moment** experienced by the beam.

Solution:

a) For  $0 \leq x \leq a$ , the slope of the line is  $m = \frac{M|_{x=2.067} - M|_{x=0}}{2.067 - 0} = \frac{64.3 - 0}{2.067 - 0} \approx 31.11$ , so

$$M(x) = mx = 31.11x \quad (1)$$

For  $a \leq x \leq L$ , the slope of the line is  $m = \frac{M|_{x=5} - M|_{x=4.378}}{5 - 4.378} = \frac{0 - 42.87}{5 - 4.378} \approx -68.92$

Using the point-slope form of a line, we can write

$$(M - 42.87) = -68.92(x - 4.378) \Rightarrow M = (42.87 + (68.92 \times 4.378)) - 68.92x$$

or

$$M(x) = 344.6 - 68.92x \quad (2)$$

b) The load  $P$  is located at the point  $x = a$  where moment equations (1) and (2) are equal, that is, at the **intersection** of the two lines. To find  $a$ , set

$$31.11x = 344.6 - 68.92x \Rightarrow (31.11 + 68.92)x|_{x=a} = 344.6$$

$$\Rightarrow a = \frac{344.6}{31.11 + 68.92} \approx 3.445 \text{ (ft)}$$

c) The **maximum moment** experienced by the beam can be calculated by substituting the value of  $a$  into equations (1) or (2).

$$M_{\max} = M(a) = 31.11 \times 3.445 = 107.2 \text{ (ft-lb)} \quad \text{or} \quad M_{\max} = 344.6 - (68.92 \times 3.445) = 107.2 \text{ (ft-lb)}$$

