

## Elementary Dynamics

### Acceleration Profiles

Given: A car has an acceleration profile as shown. It's initial position and velocity are zero. That is,  $s(0) = v(0) = 0$ .

$$\boxed{v(t) = \int a(t) dt} \quad \text{and} \quad \boxed{s(t) = \int v(t) dt}$$

Find: (a) The velocity function  $v(t)$ ; and  
(b) the displacement function  $s(t)$  for the car for  $0 \leq t \leq 20$  (sec).

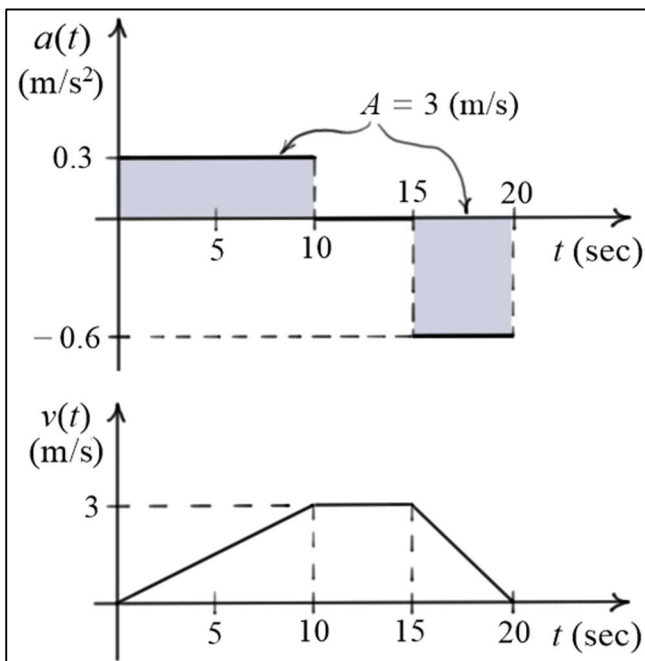
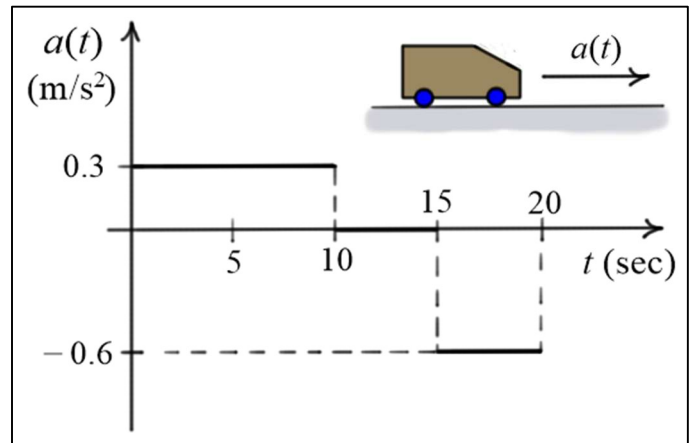
Solution:

(a) We can construct the velocity diagram easily from the acceleration diagram. When the acceleration is constant, the velocity varies linearly, and when the acceleration is zero, the velocity is constant.

$$\boxed{v(t) = \int 0.3 dt = 0.3t + D = 0.3t} \quad \text{for } 0 \leq t \leq 10 \quad (\text{recall that } v(0) = 0)$$

$$\boxed{v(t) = \int 0 dt = D = v(10) = 3 \text{ (m/s)}} \quad \text{for } 10 \leq t \leq 15$$

$$\boxed{v(t) = \int -0.6 dt = -0.6t + D = -0.6t + 12} \quad \text{for } 15 \leq t \leq 20 \quad (v(15) = 3 \text{ (m/s)})$$



The **areas** under the acceleration profile give the **changes** in velocity over those periods of time

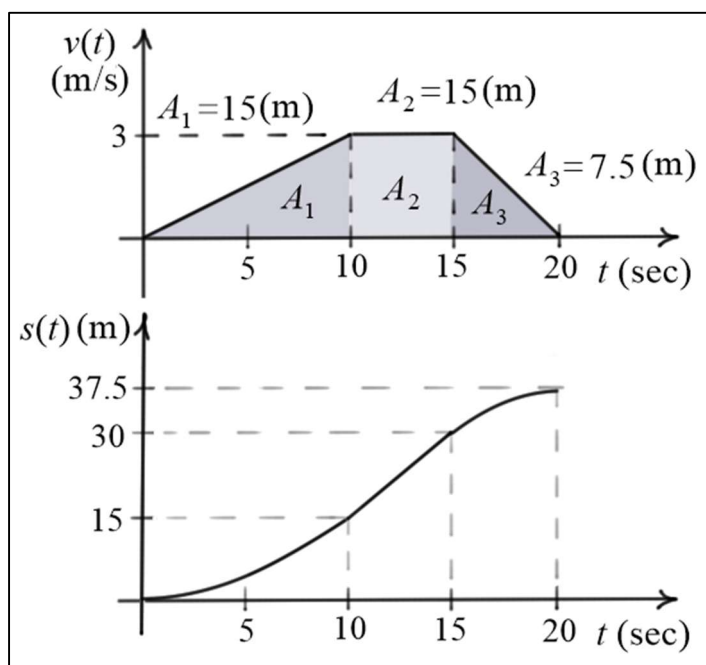
(b) The displacement function can now be derived from the velocity profile. When the velocity varies **linearly**, the displacement will vary **quadratically**, and when the velocity is **constant**, the displacement will vary **linearly**. The displacement changes are given by areas under the velocity function.

$$s(t) = \int 0.3t \, dt = 0.15t^2 + D = 0.15t^2 \quad \text{for } 0 \leq t \leq 10 \quad (\text{recall that } s(0) = 0)$$

$$s(t) = \int 3 \, dt = 3t + D = 3t - 15 \quad \text{for } 10 \leq t \leq 15 \quad (s(10) = 15 \text{ (m)})$$

$$s(t) = \int -0.6t + 12 \, dt = -0.3t^2 + 12t + D \quad \text{for } 15 \leq t \leq 20 \quad (s(15) = 30 \text{ (m)})$$

$$= -0.3t^2 + 12t - 82.5$$



The **areas** under the velocity profile give the **changes** in displacement over those periods of time

In this case, note that the acceleration profile is a **discontinuous** function, the velocity profile is a **piece-wise continuous** function, and the displacement profile is a **continuous** function. If we do not need the actual functions, we can still construct the velocity and displacement profiles by simply measuring areas under the acceleration and velocity profiles.

Plots of the position, velocity, and acceleration profiles are shown in the figure below.

