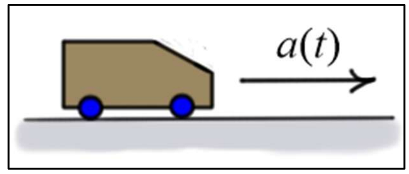
Elementary Dynamics – Example #2: (Rectilinear Motion)



Given: $a(t) = 10 - 2t^2$ (m/s^2) … the ***acceleration*** of the cart

Initial Conditions: $v(0) = 10$ (m/s) and $s(0) = 5$ (m)

Find: a) $v(t)$ … the ***velocity*** of the particle ***as a function of time***; b) $s(t)$ … the ***position*** of the particle ***as a function of time***; c) s_T … the ***total distance traveled*** from $0 \to 10$ (sec)

Solution:

a) $\boxed{\dfrac{dv}{dt} = a(t)} \Rightarrow \displaystyle\int_{v(0)}^{v(t)} dv = \int_0^t a(t)\,dt \Rightarrow v(t) - v(0) = \int_0^t \left(10 - 2t^2\right) dt$ … using definite integrals

$$\Rightarrow \quad v(t) = v(0) + \left(10t - \tfrac{2}{3}t^3\right)_0^t \quad \Rightarrow \quad \boxed{v(t) = 10 + 10t - \tfrac{2}{3}t^3} \text{ (m/s)}$$

or

$v(t) = \int a(t)\,dt = \int \left(10 - 2t^2\right) dt = 10t - \tfrac{2}{3}t^3 + D$ … using indefinite integrals

then, using the ***initial condition***

$$v(0) = 10 = \left(10t - \tfrac{2}{3}t^3 + D\right)_{t=0} = D \quad \Rightarrow \quad \boxed{v(t) = 10 + 10t - \tfrac{2}{3}t^3} \text{ (m/s)}$$

b) $s(t) = \int v(t)\,dt = \int \left(10 + 10t - \tfrac{2}{3}t^3\right) dt = 10t + 5t^2 - \tfrac{1}{6}t^4 + D$ … using indefinite integrals

then, using the ***initial condition***

$$s(0) = 5 = \left(10t + 5t^2 - \tfrac{1}{6}t^4 + D\right)_{t=0} = D \quad \Rightarrow \quad \boxed{s(t) = 5 + 10t + 5t^2 - \tfrac{1}{6}t^4} \text{ (m)}$$

c) To find the ***total distance traveled*** from $0 \to 10$ (sec), we need to determine if the particle ***changes direction*** during that time. To do this, we find the times when the ***velocity*** is ***zero***.

$$\boxed{v(t) = 10 + 10t - \tfrac{2}{3}t^3 = 0} \quad \Rightarrow \quad t = \left\{ \underbrace{-3.215}_{\times}, \underbrace{-1.085}_{\times}, \boxed{4.2998} \right\}$$

So, we can calculate the ***total distance*** traveled by first calculating the locations of the particle at the beginning and end of the interval and when the particle changes direction:

$$s(0) = 5, \quad s(4.2998) = 83.47, \text{ and } s(10) = -1061.67$$

$$\boxed{s_T = \underbrace{(83.47 - 5)}_{\substack{\text{distance from starting} \\ \text{position to zero velocity}}} + \underbrace{83.47}_{\substack{\text{distance to} \\ \text{return to origin}}} + \underbrace{1061.67}_{\substack{\text{distance from origin} \\ \text{to final position}}} = 1223.61 \approx 1220 \text{ (m)}}$$