

## Elementary Dynamics Acceleration Profiles – A Second Example

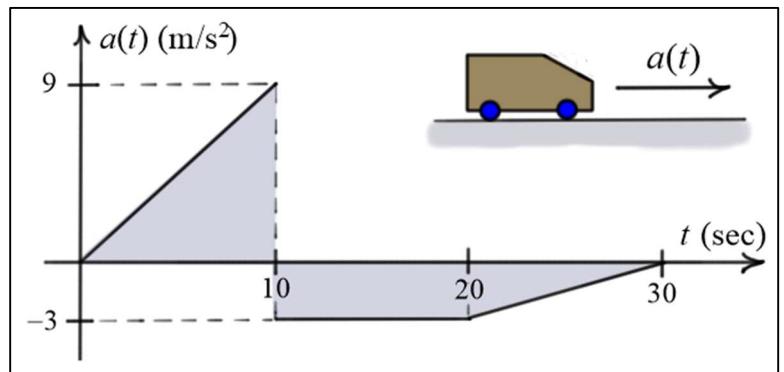
Given:

A car has the acceleration profile shown where  $a(t)$  is in  $\text{m/s}^2$ .

Find:

$v(t)$  and  $s(t)$  the speed and position of the car as functions of time for  $0 \leq t \leq 30$  (sec). Also, find the speed and position of the car when  $t = 30$  (sec).

Assume  $v(0) = s(0) = 0$ .



Solution:

$$0 \leq t \leq 10 \text{ (sec)}: \quad a(t) = 0.9t = \frac{dv}{dt}, \quad v(t) = \frac{ds}{dt}$$

$$\int_{v_0}^{v(t)} dv = \int_0^t 0.9t dt \Rightarrow \boxed{v(t) = 0.45t^2} \Rightarrow v(10) = 45 \text{ (m/sec)}$$

$$\int_{s_0}^{s(t)} ds = \int_0^t 0.45t^2 dt \Rightarrow \boxed{s(t) = 0.15t^3} \Rightarrow s(10) = 150 \text{ (m)}$$

$$10 \leq t \leq 20 \text{ (sec)}: \quad a(t) = -3 = \frac{dv}{dt}, \quad v(t) = \frac{ds}{dt}$$

$$\int_{v(10)}^{v(t)} dv = \int_{10}^t -3 dt \Rightarrow \boxed{v(t) = 75 - 3t} \Rightarrow v(20) = 15 \text{ (m/sec)}$$

$$\int_{s(10)}^{s(t)} ds = \int_{10}^t (75 - 3t) dt \Rightarrow \boxed{s(t) = -\frac{3}{2}t^2 + 75t - 450} \Rightarrow s(20) = 450 \text{ (m)}$$

$$20 \leq t \leq 30 \text{ (sec)}: \quad a(t) = 0.3t - 9 = \frac{dv}{dt}, \quad v(t) = \frac{ds}{dt}$$

$$\int_{v(20)}^{v(t)} dv = \int_{20}^t (0.3t - 9) dt \Rightarrow \boxed{v(t) = 0.15t^2 - 9t + 135} \Rightarrow \boxed{v(30) = 0 \text{ (m/sec)}}$$

$$\int_{s(20)}^{s(t)} ds = \int_{20}^t (0.15t^2 - 9t + 135) dt \Rightarrow \boxed{s(t) = 0.05t^3 - 4.5t^2 + 135t - 850} \\ \Rightarrow \boxed{s(30) = 500 \text{ (m)}}$$

Plots of the position, velocity and acceleration functions for  $0 \leq t \leq 30$  (sec) are shown below.

Note the *areas* under the *acceleration diagram* give the *changes* in the *velocity*, and the *areas* under the *velocity diagram* give the *changes* in the *position*. The acceleration diagram is discontinuous, the velocity diagram is piecewise continuous, and the position diagram is continuous.

