

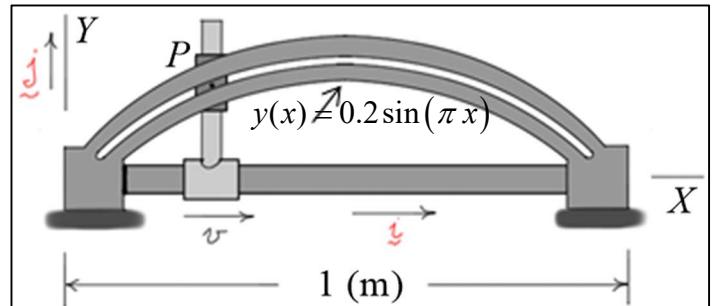
## Elementary Dynamics Example #4: (Two-Dimensional Motion, Rectangular Components)

Given:  $v_x = \dot{x} = 2 \text{ (m/s)}$  ... constant

Find:  $|\dot{y}_P|$  and  $|\ddot{a}_P|$  when  $x = 0.25 \text{ (m)}$

Solution:

To find  $\dot{y}_P$  we can **differentiate** the **position vector** of  $P$  relative to the origin.



$$r_P = x \dot{i} + y \dot{j} \Rightarrow \dot{y}_P = dr_P/dt = \dot{x} \dot{i} + \dot{y} \dot{j} \Rightarrow \begin{cases} \dot{x} = v_x = 2 \text{ (m/s)} \\ \dot{y} = \frac{d}{dt}(0.2 \sin(\pi x)) = 0.2 \cos(\pi x)(\pi \dot{x}) \end{cases}$$

At  $x = 0.25 \text{ (m)}$ ,  $\dot{y} = 0.2(2\pi) \cos\left(\frac{\pi}{4}\right) = 0.2\pi\sqrt{2} \approx 0.889 \text{ (m/s)}$

$$\Rightarrow \dot{y}_P = 2 \dot{i} + 0.889 \dot{j} \Rightarrow |\dot{y}_P| = \sqrt{2^2 + (0.2\pi\sqrt{2})^2} \approx 2.19 \text{ (m/s)}$$

To find  $\ddot{a}_P$  we can **differentiate** the **velocity vector**.

$$\dot{y}_P = dr_P/dt = \dot{x} \dot{i} + \dot{y} \dot{j} \Rightarrow \ddot{a}_P = d\dot{y}_P/dt = \ddot{x} \dot{i} + \ddot{y} \dot{j}$$

where  $\ddot{x} = 0$  and  $\ddot{y} = \frac{d}{dt}(0.2\pi\dot{x}\cos(\pi x)) = \underbrace{0.2\pi\ddot{x}\cos(\pi x)}_{\text{zero}} - 0.2(\pi\dot{x})^2\sin(\pi x)$

At  $x = 0.25 \text{ (m)}$ ,  $\ddot{y} = -0.2(\pi\dot{x})^2\sin(\pi x) = -0.2(2\pi)^2\sin\left(\frac{\pi}{4}\right) \approx 5.58 \text{ (m/s}^2)$

$$\Rightarrow |\ddot{a}_P| \approx 5.58 \text{ (m/s}^2)$$