

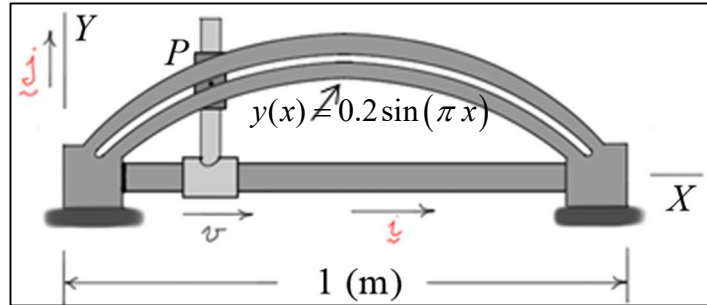
Elementary Dynamics Example #4: (Two-Dimensional Motion, Rectangular Components)

Given: $v_x = \dot{x} = 2 \text{ (m/s)}$... constant

Find: $|v_P|$ and $|a_P|$ when $x = 0.25 \text{ (m)}$

Solution:

To find v_P we can **differentiate** the **position vector** of P relative to the origin.



$$\underline{r}_P = x \underline{i} + y \underline{j} \Rightarrow \underline{v}_P = d\underline{r}_P/dt = \dot{x} \underline{i} + \dot{y} \underline{j} \Rightarrow \begin{cases} \dot{x} = v_x = 2 \text{ (m/s)} \\ \dot{y} = \frac{d}{dt}(0.2 \sin(\pi x)) = 0.2 \cos(\pi x)(\pi \dot{x}) \end{cases}$$

$$\text{At } x = 0.25 \text{ (m)}, \quad \dot{y} = 0.2(2\pi) \cos\left(\frac{\pi}{4}\right) = 0.2\pi\sqrt{2} \approx 0.889 \text{ (m/s)}$$

$$\Rightarrow \underline{v}_P = 2 \underline{i} + 0.889 \underline{j} \Rightarrow |\underline{v}_P| = \sqrt{2^2 + (0.2\pi\sqrt{2})^2} \approx 2.19 \text{ (m/s)}$$

To find a_P we can **differentiate** the **velocity vector**.

$$\underline{v}_P = d\underline{r}_P/dt = \dot{x} \underline{i} + \dot{y} \underline{j} \Rightarrow \underline{a}_P = d\underline{v}_P/dt = \ddot{x} \underline{i} + \ddot{y} \underline{j}$$

$$\text{where } \underline{\ddot{x}} = 0 \text{ and } \ddot{y} = \frac{d}{dt}(0.2\pi\dot{x}\cos(\pi x)) = \underbrace{0.2\pi\ddot{x}\cos(\pi x)}_{\text{zero}} - 0.2(\pi\dot{x})^2\sin(\pi x)$$

$$\text{At } x = 0.25 \text{ (m)}, \quad \ddot{y} = -0.2(\pi\dot{x})^2\sin(\pi x) = -0.2(2\pi)^2\sin\left(\frac{\pi}{4}\right) \approx 5.58 \text{ (m/s}^2\text{)}$$

$$\Rightarrow |\underline{a}_P| \approx 5.58 \text{ (m/s}^2\text{)}$$