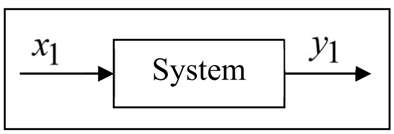
Introductory Control Systems
Linearity Principles

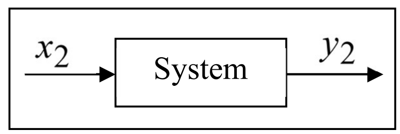
- The analysis of nonlinear systems is, in general, quite complicated – ***solutions to nonlinear equations are not easily obtained***.
- However, if these systems remain ***close*** to some ***nominal operating conditions***, then it may be appropriate to analyze their behavior using a model that has been ***linearized*** about the nominal conditions. This ***simplifies*** the analysis of the system behavior.
- ***Two properties*** must be satisfied by a system (or model) for it to be linear. These properties are defined in terms of the system input and output as follows.

1. ***Principle of Superposition***

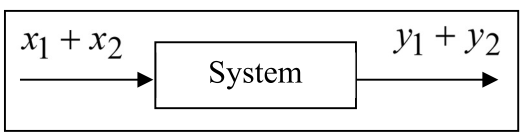
If



and



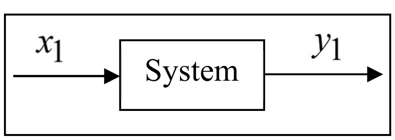
then



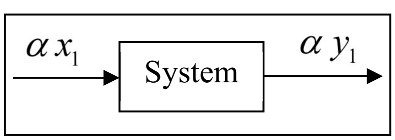
That is, if a system input x_1 produces output y_1, and a system input x_2 produces output y_2, then a system input $x_1 + x_2$ produces an output $y_1 + y_2$.

2. ***Principle of Homogeneity*** (α is a constant)

If



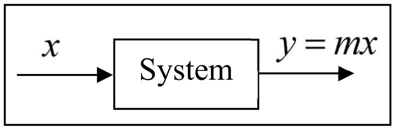
then



That is, if a system input x_1 produces output y_1, then a system input αx_1 produces an output αy_1.

Examples

1.

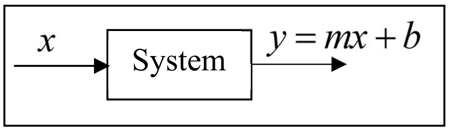


$\alpha x \to m(\alpha x) = \alpha(mx) = \alpha y$ … homogeneity satisfied

$x_1 + x_2 \to m(x_1 + x_2) = mx_1 + mx_2 = y_1 + y_2$ … superposition satisfied

System is ***linear***.

2.

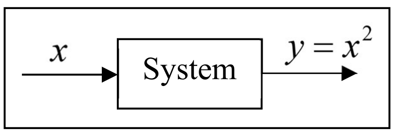


$\alpha x \to m(\alpha x) + b = \alpha(mx) + b \neq \alpha y$ … homogeneity ***not*** satisfied

$x_1 + x_2 \to m(x_1 + x_2) + b = mx_1 + mx_2 + b \neq y_1 + y_2$ … superposition ***not*** satisfied

System is ***nonlinear***.

3.

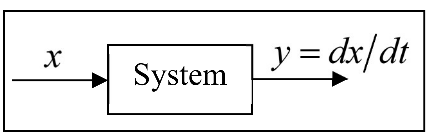


$\alpha x \to (\alpha x)^2 = \alpha^2 x^2 = \alpha^2 y \neq \alpha y$ … homogeneity ***not*** satisfied

$x_1 + x_2 \to (x_1 + x_2)^2 = x_1^2 + x_2^2 + 2x_1 x_2 \neq y_1 + y_2$ … superposition ***not*** satisfied

System is ***nonlinear***.

4.

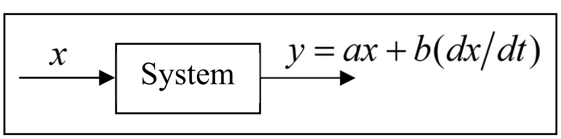


$\alpha x \to d(\alpha x)/dt = \alpha(dx/dt) = \alpha y$ … homogeneity satisfied

$x_1 + x_2 \to d(x_1 + x_2)/dt = (dx_1/dt) + (dx_2/dt) = y_1 + y_2$ … superposition satisfied

System is ***linear***.

5.



$\alpha x \to a(\alpha x) + b(d(\alpha x)/dt) + = \alpha(ax + b(dx/dt)) = \alpha y$ … homogeneity satisfied

$$\left.\begin{aligned} x_1 + x_2 &\to a(x_1 + x_2) + b(d(x_1 + x_2)/dt) \\ &= (ax_1 + b(dx_1/dt)) + (ax_2 + b(dx_2/dt)) = y_1 + y_2 \end{aligned}\right\}$$
… superposition satisfied

System is ***linear***.