

Elementary Engineering Mathematics

Application of Quadratic Equations in Electric Circuits

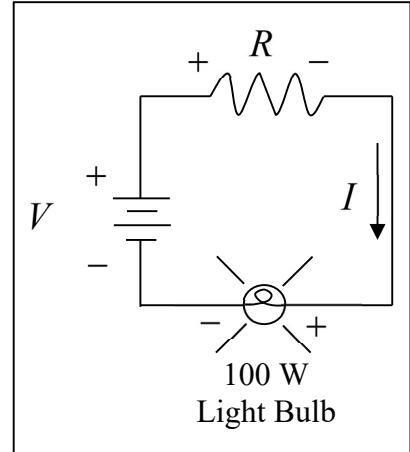
Example #1

Given: A 100-watt light bulb is connected in **series** with a resistor $R = 10$ (ohms). The applied voltage is $V = 120$ (volts).

The **power** used by the light bulb is calculated as $P_L = V_L I$. The units of power are “volt-amps” or “watts.”

Find: The current I in amperes (amps).

Solution:



To find the current, we first apply **Kirchhoff's voltage law** to the circuit.

$$V = 120 = V_R + V_L \quad (1)$$

Then, represent the voltage drops across the resistor and the light bulb in terms of the current I .

$$\text{Ohm's law: } V_R = R I = 10 I \quad (2)$$

$$\text{Power equation: } P_L = V_L I \Rightarrow V_L = P_L / I = 100 / I \quad (3)$$

Substituting from equations (2) and (3) into equation (1), gives

$$120 = (10 I) + (100 / I) \Rightarrow 120 I = 10 I^2 + 100 \Rightarrow 10 I^2 - 120 I + 100 = 0 \quad (4)$$

Quadratic formula

Using the quadratic formula, the roots of equation (4) are calculated as follows

$$I_{1,2} = \frac{120 \pm \sqrt{120^2 - (4 \times 10 \times 100)}}{2 \times 10} \approx \frac{120 \pm 101.98}{20} \approx \begin{cases} 11.1 \text{ (amps)} \\ 0.901 \text{ (amps)} \end{cases} \quad (5)$$

Completing the Square

To complete the square, we first divide equation (4) by 10: $I^2 - 12 I + 10 = 0$

$$I^2 - 12 I + \left(\frac{12}{2}\right)^2 = -10 + \left(\frac{12}{2}\right)^2 \Rightarrow I^2 - 12 I + 6^2 = -10 + 6^2$$

$$\Rightarrow (I - 6)^2 = 26 \Rightarrow I - 6 = \pm \sqrt{26} \approx \pm 5.09902$$

$$\Rightarrow I \approx 6 \pm 5.09902 \approx \begin{cases} 11.1 \text{ (amps)} \\ 0.901 \text{ (amps)} \end{cases}$$

Factoring

As the roots are not integers, it is not easy factor the quadratic equation directly. However, given the above results, we recognize that

$$I^2 - 12I + 10 \approx (I - 11.1)(I - 0.901)$$

Back to our circuit

Question: How is it that our circuit can have two different currents?

Answer: Actually, it does not. The power of the light bulb is *rated* at a *specific voltage*, so only one of the currents is correct for a given light bulb.

Let's calculate the voltage and resistance of the light bulb at each of these currents.

a) $I \approx 0.901$ (amps) $\Rightarrow V_L = P_L/I \approx 100/0.901 \approx 111$ (volts)

$$\Rightarrow R_L = V_L/I \approx 111/0.901 \approx 123 \text{ (ohms)}$$

b) $I \approx 11.1$ (amps) $\Rightarrow V_L = P_L/I \approx 100/11.1 \approx 9.01$ (volts)

$$\Rightarrow R_L = V_L/I \approx 9.01/11.1 \approx 0.812 \text{ (ohms)}$$

Summary @ $V = 120$ (volts) and $R = 10$ (ohms)

I (amps)	$V_R = RI$ (volts)	$P_R = V_R I$ (watts)	$V_L = V - V_R$ (volts)	$R_L = V_L/I$ (ohms)	$P_L = V_L I$ (watts)
0.901	9.01	8.12	111	123	100
11.1	111	1232	9	0.812	100

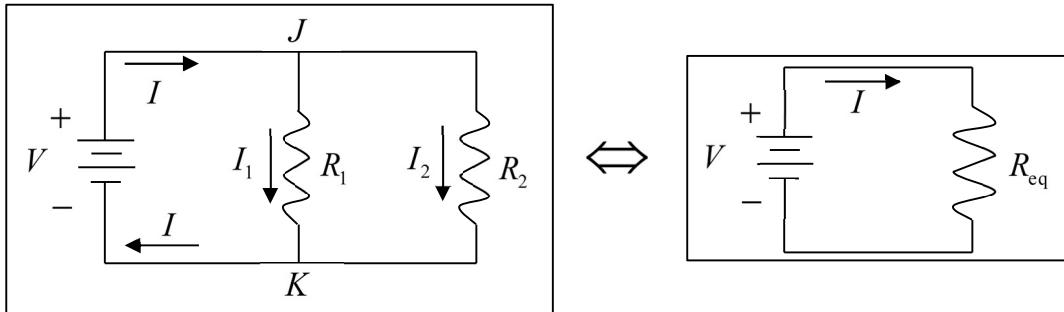
Note: The resistance of an average 100-watt light bulb at 120 volts is about 140 ohms.

Example #2

Given: The electric circuit shown below has two resistors connected in *parallel*. At node J the current I splits into two parts, I_1 and I_2 , and at node K the currents recombine to form I . The splitting and combining of currents at nodes obey **Kirchhoff's Current Law**.

Kirchhoff's Current Law:

The **sum** of the currents *flowing into* a node *equals* to the sum of the currents *flowing away* from a node. (In this case, $I = I_1 + I_2$)



Equivalent Electrical Circuits

Using Kirchhoff's law, it can be shown that the two parallel resistors (R_1 and R_2) act as a single **equivalent** resistor R_{eq} .

$$R_{\text{eq}} = \frac{R_1 R_2}{R_1 + R_2} \quad (6)$$

Finally, it is known that $R_{\text{eq}} = 150$ (ohms) and $R_2 = R_1 + 200$.

Find: The resistances R_1 and R_2 .

Solution:

Substituting the known information into equation (6) gives

$$R_{\text{eq}} = 150 = \frac{R_1 R_2}{R_1 + R_2} = \frac{R_1 (R_1 + 200)}{R_1 + (R_1 + 200)} = \frac{R_1^2 + 200R_1}{2R_1 + 200}$$

Clearing fractions and combining terms gives

$$150(2R_1 + 200) = R_1^2 + 200R_1 \Rightarrow R_1^2 + \underbrace{(200 - 300)}_{-100} R_1 - \underbrace{(150 \times 200)}_{30000} = 0$$

Using the quadratic formula, we find

$$R_1 = \frac{100 \pm \sqrt{100^2 - 4(-30000)}}{2} \approx \frac{100 \pm 360.55}{2} \approx \begin{cases} 230.28 \\ -130.28 \end{cases}$$

As the value of resistance cannot be negative, we find $R_1 \approx 230$ (ohms) and $R_2 \approx 430$ (ohms).