

Elementary Dynamics Example #5: (Rectangular Components, Projectile Motion)

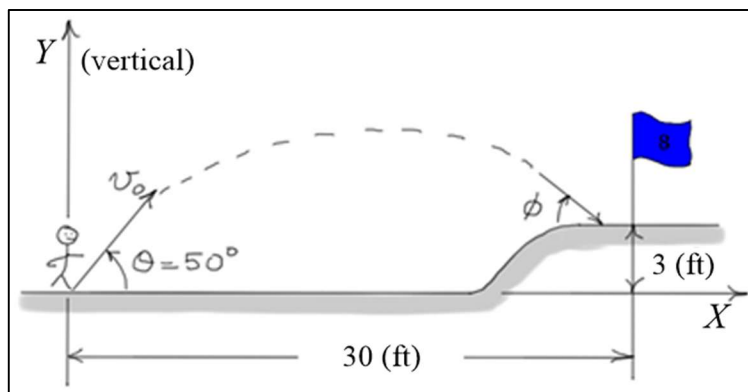
Given: neglect air resistance

initial velocity v_0 at $\theta = 50$ (deg)

ball lands 3 feet short of the hole
at (27,3) (ft)

Find: v_0 and angle ϕ at which the ball strikes the ground

Solution:



X direction: $x(t) = x_0 + v_{x_0} t = (v_0 \cos(50))t$ constant velocity in X direction

Y direction: $y(t) = y_0 + v_{y_0} t - \frac{1}{2} g t^2 = (v_0 \sin(50))t - \frac{1}{2} g t^2$ constant acceleration in Y direction

When the **ball strikes the ground**:

$$x = 27 = v_0 \cos(50)t \Rightarrow t = 27 / (v_0 \cos(50))$$

$$y = 3 = (v_0 \sin(50))t - \frac{1}{2} g t^2 = (v_0 \sin(50))27 / (v_0 \cos(50)) - \frac{1}{2} g (27 / (v_0 \cos(50)))^2$$

$$= 27 \tan(50) - \frac{27^2 g}{2 v_0^2 \cos^2(50)} \Rightarrow \frac{27^2 g}{2 v_0^2 \cos^2(50)} = 27 \tan(50) - 3$$

$$\Rightarrow v_0 = \sqrt{\frac{27^2 g}{2 \cos^2(50) (27 \tan(50) - 3)}} \approx 31.2 \text{ (ft/s)} \quad \dots \text{ initial velocity}$$

At what angle does the ball strike the ground? The **velocity vector** is **tangent** to the path, so we can use the X and Y components of the velocity of the ball when it strikes the ground.

$$v_x = v_{x_0} = v_0 \cos(50) \approx 20.056 \text{ (ft/s)} \quad (\text{constant})$$

$$v_y^2 = v_{y_0}^2 - 2g\Delta y = (v_0 \sin(50))^2 - 2g(3) \Rightarrow v_y = \sqrt{(v_0 \sin(50))^2 - 2g(3)} \approx 19.445 \text{ (ft/s)}$$

$$\phi = \tan^{-1} \left(\frac{v_y}{v_x} \right) \approx 44.1 \text{ (deg)}$$

