

Elementary Dynamics Example #7: (2D Motion, Normal & Tangential Components)

Given: $R = 200$ (ft), $d = 12$ (ft)

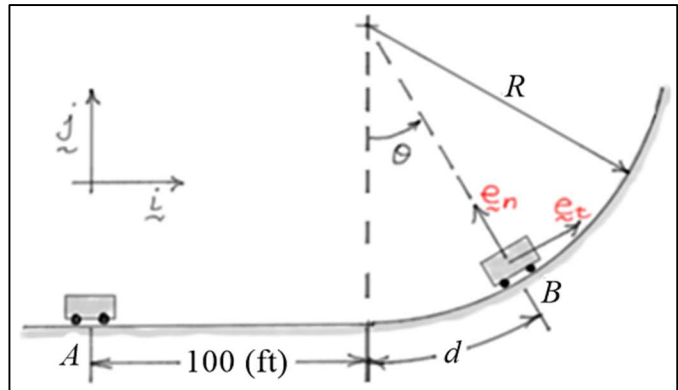
car starts from rest at A

car accelerates at a constant rate of

$$\dot{v} = a_t = 10 \text{ (ft/s}^2\text{) from } A \text{ to } B$$

Find: \underline{v}_B and \underline{a}_B in ft/s and ft/s² using **normal** and **tangential** components.

Solution:



Velocity: (given constant acceleration along the path)

$$\underline{v}_B = \sqrt{v_A^2 + 2a_t \Delta s} = \sqrt{2(10)(112)} \approx 47.3286 \approx 47.3 \text{ (ft/s)} \Rightarrow \underline{v}_B = 47.3 \underline{e}_t \text{ (ft/s)}$$

Acceleration:

$$\underline{a}_t = \dot{v} = 10 \text{ (ft/s}^2\text{)} \quad \underline{a}_n = \frac{v^2}{\rho} = \frac{2(10)(112)}{200} = 11.2 \text{ (ft/s}^2\text{)}$$



$$\underline{a}_B = 10 \underline{e}_t + 11.2 \underline{e}_n \text{ (ft/s}^2\text{)} \Rightarrow |\underline{a}_B| = \sqrt{a_t^2 + a_n^2} \approx 15.0147 \approx 15.0 \text{ (ft/s}^2\text{)}$$

Aside:

We could also express these results in terms of the unit vectors \underline{i} and \underline{j} . For example,

$$\theta = d/R = (12/200)(180/\pi) \approx 3.43775 \approx 3.44 \text{ (deg)}$$

$$\underline{v}_B = 47.3 \underline{e}_t = 47.3(\cos(\theta) \underline{i} + \sin(\theta) \underline{j}) = 47.2 \underline{i} + 2.84 \underline{j} \text{ (ft/s)}$$

The acceleration could also be expressed in terms of the unit vectors \underline{i} and \underline{j} by noting

that: $\underline{e}_n = -\sin(\theta) \underline{i} + \cos(\theta) \underline{j}$.