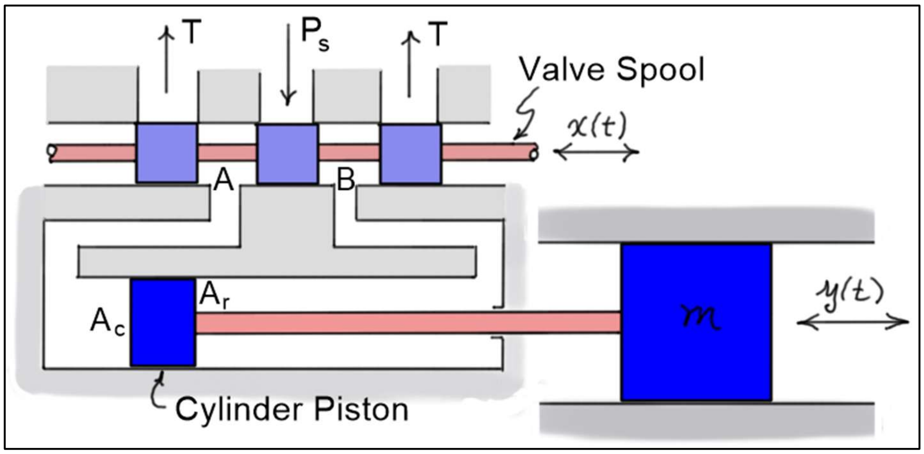
Introductory Control Systems
Hydraulic Positioning System

Positioning System – Definition of Terms

- Incompressible fluid
- A_c = cap end piston area
- A_r = rod end piston area
- m = mass of load
- b = damping coefficient
- P_s = constant supply pressure
- P = pressure on the piston
- p = ΔP, the change in P
- X = valve spool position
- x = ΔX, the change in X
- Y = load position
- y = ΔY, the change in Y



Operation

- If $X > 0$, the ***pressure source*** is applied to the A port of the valve and the ***cap end*** of the cylinder causing the load to ***move right***. Return flow to the tank is though the B port.
- If $X < 0$, the ***pressure source*** is applied to the B port of the valve and the ***rod end*** of the cylinder causing the load to ***move left***. Return flow to the tank is though the A port.

Flow Model

If $X > 0$, the pressure source is applied to the A port of the valve. As a result, fluid flows into the piston chamber. The ***volumetric flow rate*** Q through the valve is a function of the spool position X and the pressure P in the piston chamber.

$$Q = g(X, P) \tag{1}$$

To simplify the model, Eq. (1) can be ***linearized*** about some operational (set) point (X_0, P_0). This is done using a ***Taylor series expansion*** as discussed in earlier notes. The ***change*** in ***flow rate*** can be written as

$$q \triangleq \Delta Q = \left(\frac{\partial g}{\partial X}\right)_{X_0,P_0} \Delta X + \left(\frac{\partial g}{\partial P}\right)_{X_0,P_0} \Delta P \qquad (2)$$
$$= (k_x)x - (k_p)p$$

where k_x and k_p represent the ***derivatives*** of the function $g(X,P)$ with respect to X and P, respectively. The minus sign in the second of equations (2) indicates the flow rate ***decreases*** as the ***pressure*** in the piston chamber ***increases***.

Assuming the fluid is ***incompressible***, the ***volumetric flow rate*** can be related to the ***speed*** of the piston as follows.

$$Q = A_c\dot{Y} \qquad (3)$$

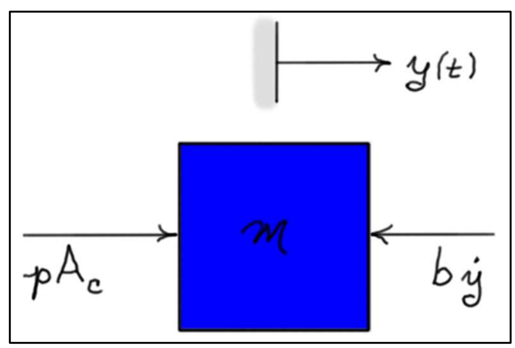
Letting $Q = Q_0 + q$, $\dot{Y} = \dot{Y}_0 + \dot{y}$, and $Q_0 = A_c\dot{Y}_0$, then ***changes*** in the ***volumetric flow rate*** can be related to ***changes*** in the ***speed*** of the piston as follows.

$$q = A_c\dot{y} \qquad (4)$$

Combining Eqs. (4) and (2) gives a relationship between the changes in pressure, valve spool position, and speed of the piston.

$$\boxed{p = (k_x x - A_c\dot{y})/k_p} \qquad (5)$$

Model for Piston Movement



Assuming the pressure on the rod end of the piston is small compared to the pressure on the cap end, Newton's law gives

$$\underset{+}{\rightarrow}\sum F = p\,A_c - b\,\dot{y} = m\,\ddot{y}$$

Note that it is assumed here that the ***nominal velocity*** is ***constant***, and the ***nominal pressure*** and ***damping forces*** cancel from the force summation. Hence, the force summation represents changes from the nominal, constant velocity condition.

Rearranging the equation, and substituting for the pressure from Eq. (5) gives

$$\boxed{m\ddot{y} + \left(b + \frac{A_c^2}{k_p}\right)\dot{y} = A_c\left(\frac{k_x}{k_p}\right)x} \qquad (X > 0) \tag{6}$$

If $X < 0$, then $p = (A_r\dot{y} - k_x x)/k_p$ and $\underset{+}{\rightarrow}\sum F = -p\,A_r - b\,\dot{y} = m\,\ddot{y}$. In this case, the model equation is

$$\boxed{m\ddot{y} + \left(b + \frac{A_r^2}{k_p}\right)\dot{y} = A_r\left(\frac{k_x}{k_p}\right)x} \qquad (X < 0) \tag{7}$$

Note here that x and y are still measured ***positive*** to the ***right***.

Notes:

- Because the piston areas A_c and A_r are not equal, Eqs. (6) and (7) represent ***two different*** dynamic responses. The hydraulic cylinder will respond differently in extension and retraction.
- Conversely, if the cylinder is a ***double-rod cylinder*** with $A_c = A_r$, then the same model applies in both directions. Extension and retraction dynamics will be identical.
- The motions described by Eqs. (6) and (7) are ***second-order***, ***over-damped*** responses.
- If the mass of the load is small $(m \approx 0)$, then the response is ***first order***.