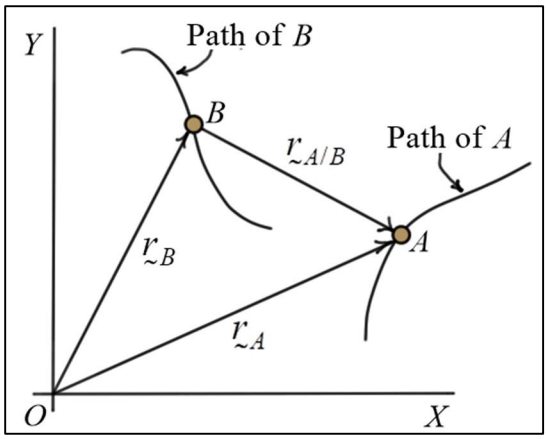
Elementary Dynamics
Relative Motion of Two Particles

The figure shows the paths of motion of two particles A and B. The vectors $\underset{\sim}{r}_A$ and $\underset{\sim}{r}_B$ represent the ***position vectors*** of A and B relative to a fixed point O, and the vector $\underset{\sim}{r}_{A/B}$ represents the position vector of A ***relative*** to B (or A with respect to B). Note that $\underset{\sim}{r}_{A/B}$ starts at B and ends at A.



Sometimes it is convenient to express the motion of a point relative to another moving point. For example, to describe the motion of particle A relative to particle B, first note that

$\underset{\sim}{r}_A = \underset{\sim}{r}_B + \underset{\sim}{r}_{A/B}$ or $\boxed{\underset{\sim}{r}_{A/B} = \underset{\sim}{r}_A - \underset{\sim}{r}_B}$.

Differentiating this expression gives rise to the definitions of the terms "***relative velocity***" and "***relative acceleration***."

$\boxed{\underset{\sim}{v}_{A/B} = \dot{\underset{\sim}{r}}_{A/B} = \underset{\sim}{v}_A - \underset{\sim}{v}_B}$ "relative velocity – velocity of A relative to B"

$\boxed{\underset{\sim}{a}_{A/B} = \ddot{\underset{\sim}{r}}_{A/B} = \underset{\sim}{a}_A - \underset{\sim}{a}_B}$ "relative acceleration – acceleration of A relative to B"

In words, the motion (velocity or acceleration) of A relative to B represents the ***motion*** of A as seen by an ***observer translating*** with B. This concept is used extensively in the analysis of rigid body kinematics.