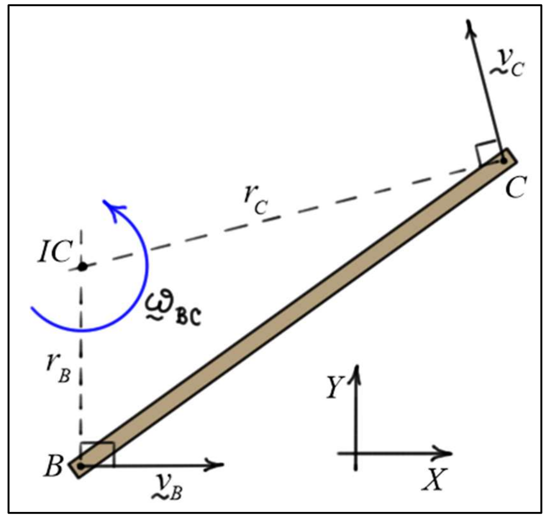
Elementary Engineering Mathematics
Application of Geometry/Trigonometry – Elementary Dynamics



$$\omega_{BC} = \frac{v_B}{r_B} = \frac{v_C}{r_C} \text{ (radians/sec)}$$

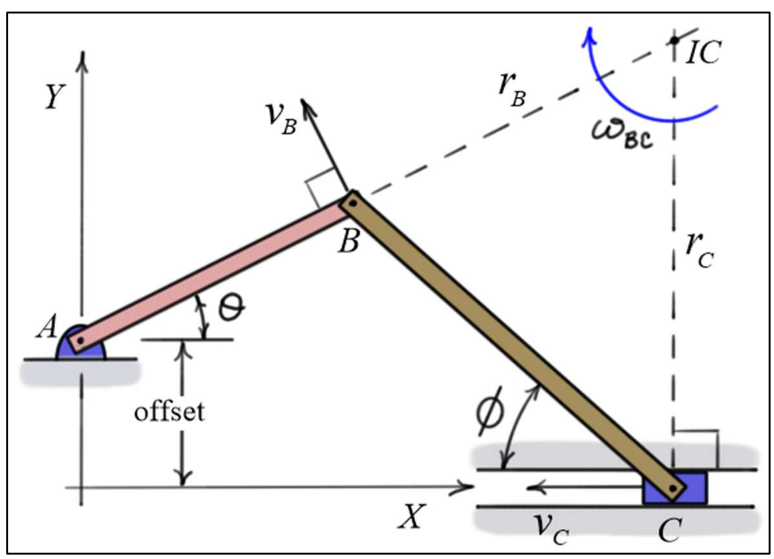
The ***two-dimensional motion*** of a rigid body at any instant of time as it moves in the XY plane can be described as ***pure rotational motion*** about an ***instantaneous center*** (IC) of ***zero velocity***. The ***location*** of the IC relative to the body (at that instant) can be found by constructing lines ***perpendicular*** to the velocities of two points on the body. The ***intersection*** of these two lines (shown as dashed lines in the figure) is the location of the instantaneous center IC. Note from instant to instant, the IC ***changes location*** relative to the body.

The scalar ***angular velocity*** of the body ω_{BC} (i.e. how fast it is rotating about the IC in radians/second) is related to the scalar ***velocities*** of the two points as shown.

Example: Slider-crank mechanism

A slider-crank mechanism with an ***offset*** is shown in the diagram. Bar AB is the ***crank***, piston C is the ***slider***, and bar BC is the ***connecting rod***. In the position shown, as crank AB rotates ***counterclockwise***, the slider moves to the ***left***. The velocity of B is perpendicular to AB, and the velocity of C is to the left along the slot (X-axis).

The ***instantaneous center*** (IC) of connecting rod BC at ***this instant*** is found by constructing the dashed lines ***perpendicular*** to the ***velocities*** of points B and C. One of these lines is ***along*** crank AB and the other is ***perpendicular*** to the slot at C. The ***intersection*** point of these two lines is the ***instantaneous center***.



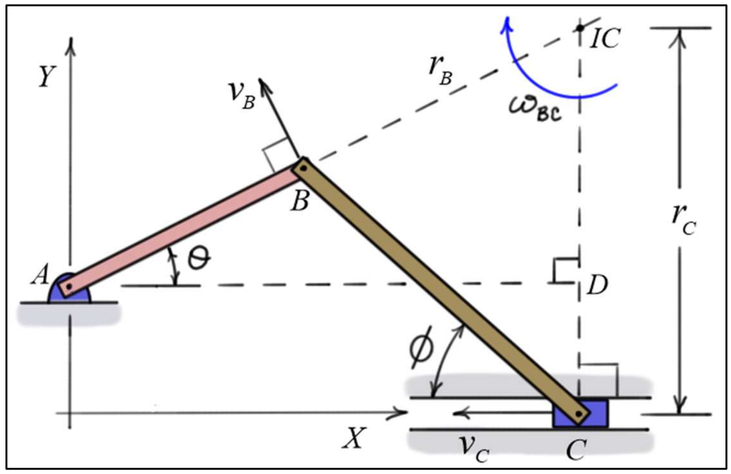
Problem:

Given: The ***coordinates*** of points *A*, *B*, and *C* (in inches) and the ***velocity*** of *B*:

A: (0, 3)   *B*: (4, 5)   *C*: (8,0)

$\boxed{v_B = 5 \text{ (in/s)}}$ in direction shown

Find: the ***location*** of *IC* and v_C the ***velocity*** of point *C* at this instant.



Solution #1: (using right triangles)

a) First, construct the dashed lines to the ***instantaneous center***. Then ***construct*** the ***right triangle*** *ADIC*, and ***calculate*** the angle of *AB* relative to *AD*.

$$\theta = \tan^{-1}\left(\frac{y_B - y_A}{x_B - x_A}\right) = \tan^{-1}\left(\frac{5-3}{4-0}\right) = \tan^{-1}\left(\frac{2}{4}\right) \approx 26.565^\circ$$

b) ***Calculate*** the distances r_B and r_C.

$$\tan(\theta) = \frac{r_C - L_{CD}}{L_{AD}} \quad \Rightarrow \quad r_C = L_{CD} + \left(L_{AD}\tan(\theta)\right) = 3 + \left(8 \times \tfrac{2}{4}\right) = 7 \text{ (in)}$$

$$\cos(\theta) = \frac{L_{AD}}{r_B + L_{AB}} \quad \Rightarrow \quad r_B = \left(\frac{L_{AD}}{\cos(\theta)}\right) - L_{AB} \approx \left(\frac{8}{\cos(26.565)}\right) - \sqrt{4^2 + 2^2} \approx 4.47214 \approx 4.47 \text{ (in)}$$

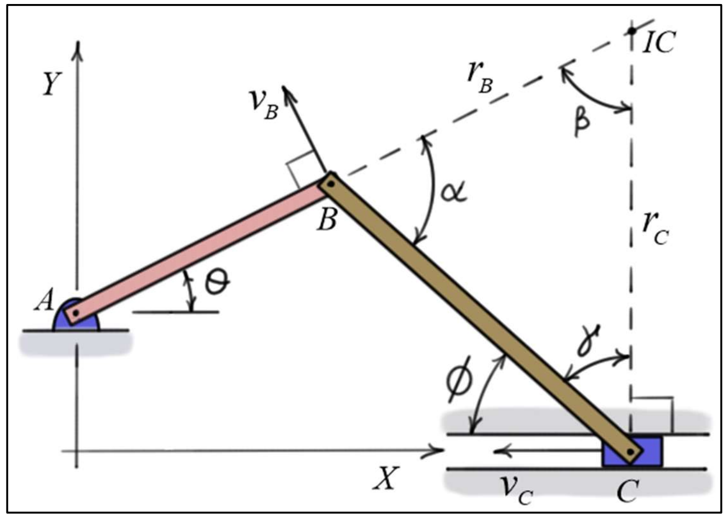
c) Find the ***angular velocity*** of *BC* and the ***velocity*** of piston *C*.

$$\omega_{BC} = \frac{v_B}{r_B} \approx \frac{5 \text{ (in/s)}}{4.47214 \text{ (in)}} \approx 1.118 \approx 1.12 \text{ (rad/sec)}$$ (angular motion is ***clockwise***)

$$\frac{v_B}{r_B} = \frac{v_C}{r_C} \quad \Rightarrow \quad v_C = \left(\frac{r_C}{r_B}\right) v_B \approx \left(\frac{7}{4.47214}\right) 5 \approx 7.82624 \approx 7.83 \text{ (in/s)}$$

Note: When analyzing slider-crank mechanisms, we have the ***advantage*** of being able to use ***right triangles***; however, for more complex mechanisms (such as ***four-bar*** mechanisms), we will often need a more ***general approach*** using ***non-right triangles***.

Solution #2: (using non-right triangles)



a) First, calculate the angles of *AB* and *BC* relative to the *X*-axis.

$$\theta = \tan^{-1}\left(\frac{y_B - y_A}{x_B - x_A}\right) = \tan^{-1}(2/4) \approx 26.565^\circ$$

$$\phi = \tan^{-1}\left(\frac{y_B - y_C}{x_C - x_B}\right) = \tan^{-1}(5/4) \approx 51.34^\circ$$

b) Construct the dashed lines to the ***instantaneous center***. In the newly constructed triangle *BCIC*, define the ***unknown angles*** α, β, and γ.

c) ***Calculate*** the ***unknown angles*** using the concepts from geometry.

$\alpha = \theta + \phi \approx 77.905^\circ$ $\beta = 90 - \theta \approx 63.435^\circ$ $\gamma = 90 - \phi \approx 38.66^\circ$ … ***why?***

d) Now use the ***law of sines*** to find the distances r_B and r_C.

$$\frac{\sin(\beta)}{b} = \frac{\sin(\gamma)}{r_B} \Rightarrow \frac{\sin(63.435)}{\sqrt{4^2+5^2}} = \frac{\sin(38.66)}{r_B} \Rightarrow r_B = \frac{\sin(38.66)}{\sin(63.435)}\sqrt{4^2+5^2}$$

$$\Rightarrow r_B \approx 4.47215 \approx 4.47 \text{ (inches)}$$

$$\frac{\sin(\beta)}{b} = \frac{\sin(\alpha)}{r_C} \Rightarrow \frac{\sin(63.435)}{\sqrt{4^2+5^2}} = \frac{\sin(77.905)}{r_C} \Rightarrow r_C = \frac{\sin(77.905)}{\sin(63.435)}\sqrt{4^2+5^2}$$

$$\Rightarrow r_C \approx 7.0 \text{ (inches)}$$

e) Find the ***angular velocity*** of *BC* and the ***velocity*** of piston *C* as shown above.