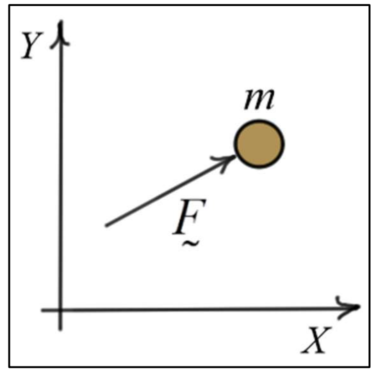
Elementary Dynamics
Kinetics of Particles: Newton's Second Law

Until now we have been studying ***kinematics***, that is, the study of motion without regard to the forces that cause (or result from) the motion. Now we want to extend our study to include the forces acting on the particle. This is called ***kinetics***. Forces may be related to motion using Newton's second law.

Newton's Second Law

Newton's second law relates the ***kinematics*** of a particle or rigid body to the ***forces*** applied to it. For a single particle, Newton's second law states

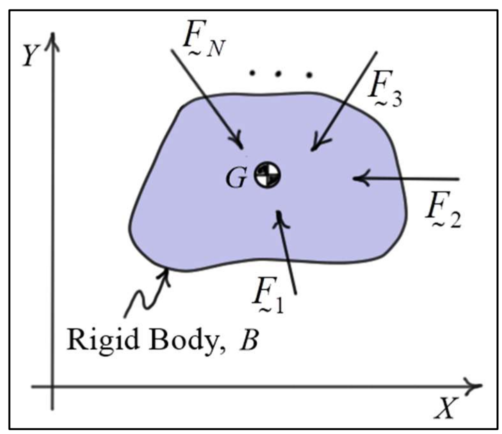


$$\boxed{\underset{\sim}{F} = m\underset{\sim}{a}}$$

Here, $\underset{\sim}{F}$ is the ***resultant force*** acting on the particle, m is its ***mass***, and $\underset{\sim}{a}$ is the ***acceleration*** of the particle measured relative to an ***inertial*** reference frame.

For a rigid body B, Newton's second law is

$$\boxed{\sum_{i=1}^{N} \underset{\sim}{F}_i = m\underset{\sim}{a}_G}$$



Here, $\underset{\sim}{F}_i \left(i = 1,\ldots,N\right)$ represent all of the ***external*** forces acting on B, m is the mass of B, and $\underset{\sim}{a}_G$ is the acceleration of G the mass-center of B measured relative to an inertial reference frame. Note that some of the external forces may ***cause motion*** to occur and some may be ***reaction*** forces.

If B is undergoing ***translational*** motion, then the above equation describes the complete motion of B. In this case, the shape of B is only important in that it determines the location of the mass-center G. However, if the body is ***rotating as well as translating***, then additional equations are needed to describe its rotational motion. These additional equations depend on the shape of B as measured by its ***moments of inertia***. When the shape of a body is not important, we consider it to be a particle.

Component Forms of Newton's Second Law

For a rigid body ***translating*** in three dimensions, we can express Newton's second law as a set of ***three scalar equations***. These equations take on different forms depending on which set of components (unit vectors) we choose. As usual, we can choose the set of components that make the problem at hand easier to solve. The best component directions to use for a specific problem are those that make the force and acceleration components have the simplest form.

Cartesian (Rectangular) Components

$$\sum_{i=1}^{N} F_{x_i} = m a_{G_x} \qquad \sum_{i=1}^{N} F_{y_i} = m a_{G_y} \qquad \sum_{i=1}^{N} F_{z_i} = m a_{G_z}$$

Normal, Tangential, and Binormal Components

$$\sum_{i=1}^{N} F_{n_i} = m a_{G_n} \qquad \sum_{i=1}^{N} F_{t_i} = m a_{G_t} \qquad \sum_{i=1}^{N} F_{b_i} = m a_{G_b} = 0$$

Cylindrical Components

$$\sum_{i=1}^{N} F_{r_i} = m a_{G_r} \qquad \sum_{i=1}^{N} F_{\theta_i} = m a_{G_\theta} \qquad \sum_{i=1}^{N} F_{z_i} = m a_{G_z}$$

Free-Body Diagrams

Before the above component equations can be written for a specific problem, we must draw an accurate ***free-body diagram***. This diagram must contain ***all forces acting on the body*** relative to the component directions shown. ***This step is essential to a meaningful and accurate solution.***