

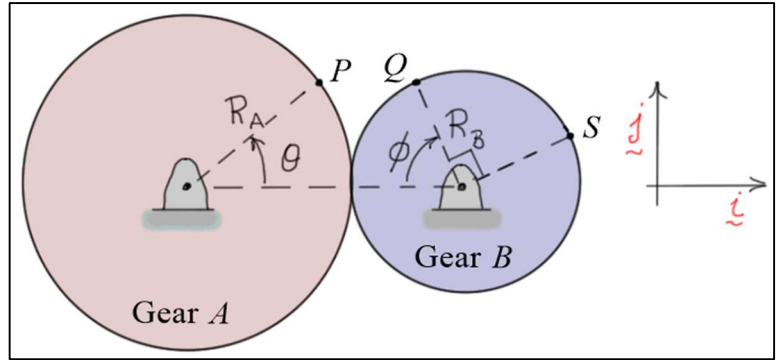
## Elementary Dynamics Example #12: (2D Motion, Relative Motion)

Given:  $R_A = 3$  (in),  $R_B = 2$  (in)

$\omega_A = \dot{\theta} = 1000$  (rpm) ... constant

$\omega_B = \dot{\phi} = 1500$  (rpm) ... constant

➤ when  $\theta = \phi = 0$ ,  $P$  and  $Q$  are touching, and  $S$  is directly above the support.



Find:  $\underline{v}_{Q/P}$ ,  $\underline{a}_{Q/P}$ ,  $\underline{v}_{S/P}$ ,  $\underline{a}_{S/P}$  at  $\theta = \phi = 0$

Solution:

$$\left. \begin{aligned} \underline{v}_P &= R_A \dot{\theta} \underline{j} = \left(\frac{3}{12}\right) \left(1000 \left(\frac{2\pi}{60}\right)\right) \underline{j} \approx 26.1799 \underline{j} \approx 26.2 \underline{j} \text{ (ft/s)} \\ \underline{v}_Q &= R_B \dot{\phi} \underline{j} = \left(\frac{2}{12}\right) \left(1500 \left(\frac{2\pi}{60}\right)\right) \underline{j} \approx 26.1799 \underline{j} \approx 26.2 \underline{j} \text{ (ft/s)} \end{aligned} \right\} \underline{v}_{Q/P} = \underline{v}_Q - \underline{v}_P = 0$$

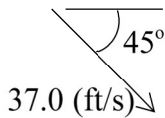
Note: for gears in contact,  $R_A \dot{\theta} = R_B \dot{\phi}$  where  $R_A$  and  $R_B$  represent the **pitch radii** of the two gears.

$$\left. \begin{aligned} \underline{a}_P &= -\left(R_A \dot{\theta}^2\right) \underline{i} = -\left(\frac{3}{12}\right) \left(1000 \left(\frac{2\pi}{60}\right)\right)^2 \underline{i} \approx -2741.56 \underline{i} \text{ (ft/s}^2\text{)} \\ \underline{a}_Q &= \left(R_B \dot{\phi}^2\right) \underline{i} = \left(\frac{2}{12}\right) \left(1500 \left(\frac{2\pi}{60}\right)\right)^2 \underline{i} \approx 4112.34 \underline{i} \text{ (ft/s}^2\text{)} \end{aligned} \right\} \underline{a}_{Q/P} = \underline{a}_Q - \underline{a}_P \approx 6850 \underline{i} \text{ (ft/s}^2\text{)}$$

So, the **relative acceleration** of the two contacting points is **not zero**.

$$\underline{v}_S = R_B \dot{\phi} \underline{i} = \left(\frac{2}{12}\right) \left(1500 \left(\frac{2\pi}{60}\right)\right) \underline{i} \approx 26.1799 \underline{i} \approx 26.2 \underline{i} \text{ (ft/s)}$$

$$\underline{v}_{S/P} = \underline{v}_S - \underline{v}_P = 26.2 \underline{i} - 26.2 \underline{j} \text{ (ft/s)}$$



Finally,

$$\underline{a}_S = -\left(R_B \dot{\phi}^2\right) \underline{j} = -\left(\frac{2}{12}\right) \left(1500 \left(\frac{2\pi}{60}\right)\right)^2 \underline{j} \approx -4112.34 \underline{j} \text{ (ft/s}^2\text{)}$$

$$\underline{a}_{S/P} = \underline{a}_S - \underline{a}_P \approx 2740 \underline{i} - 4110 \underline{j} \text{ (ft/s}^2\text{)}$$

