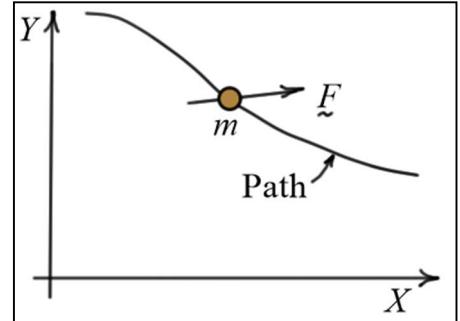


Elementary Dynamics

Principle of Impulse and Momentum for Particles

Recall that Newton's second law for a particle can be written as $\mathbf{F} = m\mathbf{a}$, where \mathbf{F} is the **resultant** force acting on the particle and \mathbf{a} is its **acceleration**. Recall also that the force and the acceleration are generally not tangent to the path, except in the case of rectilinear motion.



Previously, we developed an **integrated scalar form** of Newton's second law, namely the **principle of work and energy**. Here we develop an **integrated vector form** of Newton's second law called the **principle of impulse and momentum**. Integrating Newton's law with respect to **time** gives:

$$I_{1 \rightarrow 2} \triangleq \int_{t_1}^{t_2} \mathbf{F} dt = m \int_{t_1}^{t_2} \left(\frac{d\mathbf{v}}{dt} \right) dt = m\mathbf{v}_2 - m\mathbf{v}_1 = \Delta \mathbf{v} \quad \text{or}$$

$$m\mathbf{v}_1 + \int_{t_1}^{t_2} \mathbf{F} dt = m\mathbf{v}_2$$

This is the **principle of linear impulse and momentum**. It states that the **resultant linear impulse** on the system over the **time interval** from $t_1 \rightarrow t_2$ gives rise to a **change** in **linear momentum**. Note that this equation, like Newton's law, is a **vector** equation. Also, note that the term "**linear**" means "**translational**" as opposed to "**angular**" or "**rotational**".

This principle can be extended to a system of particles by noting that the **total** impulse of all equal and opposite internal forces in a system is **zero**. So, in this case, for a system of particles, we have

$$\sum m_i (\mathbf{v}_i)_1 + \sum \int_{t_1}^{t_2} \mathbf{F}_i dt = \sum m_i (\mathbf{v}_i)_2 \quad \text{or}$$

$$m(\mathbf{v}_G)_1 + \sum \int_{t_1}^{t_2} \mathbf{F}_i dt = m(\mathbf{v}_G)_2$$

Here, $\sum \int_{t_1}^{t_2} \mathbf{F}_i dt$ represents the **sum** of the impulses of all **external forces** and **internal forces** acting on the system, and G represents the **mass center** of the system. The **time-averaged, net force** acting on the system is defined to be

$$\boxed{\tilde{F}_{avg} = \frac{1}{\Delta t} \sum \int_{t_1}^{t_2} \tilde{F}_i \, dt = \frac{1}{\Delta t} I_{1 \rightarrow 2}}$$

Definitions: Forces that are large, act over a short interval of time, and cause *significant* changes in momentum are called *impulsive* forces. Forces that cause *insignificant* changes in momentum over short intervals of time are called *non-impulsive* forces. *Weight forces* are generally considered to be *non-impulsive*.