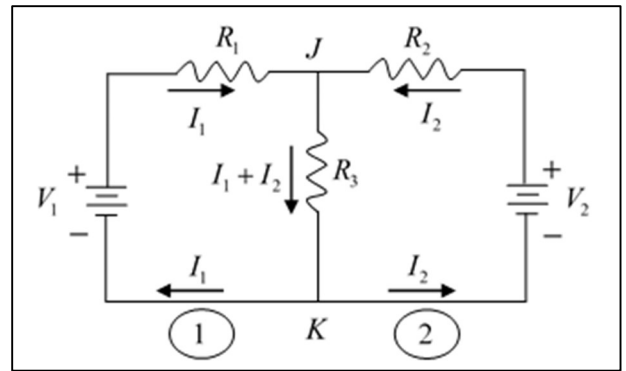# Elementary Engineering Mathematics
Applications of Systems of Linear, Algebraic Equations in Electrical Engineering

Consider the double-loop DC circuit shown in the diagram. Application of **Kirchhoff's Current Law** tells us that the current in the middle path (common to both loops) is the **sum** of the currents flowing in the outer paths. Using this result, we apply **Kirchhoff's Voltage Law** to each loop separately.

$$\sum_{\text{loop 1}} \left(\text{voltage rises} \right) = V_1 = \sum_{\text{loop 1}} \left(\text{voltage drops} \right) = R_1 I_1 + R_3 \left(I_1 + I_2 \right)$$

$$\sum_{\text{loop 2}} \left(\text{voltage rises} \right) = V_2 = \sum_{\text{loop 2}} \left(\text{voltage drops} \right) = R_2 I_2 + R_3 \left(I_1 + I_2 \right)$$

or

$$\boxed{\begin{aligned} \left(R_1 + R_3 \right) I_1 + \left(R_3 \right) I_2 &= V_1 \\ \left(R_3 \right) I_1 + \left(R_2 + R_3 \right) I_2 &= V_2 \end{aligned}}$$

(1)

Given the resistances and applied voltages, Eqs. (1) can be solved for the currents I_1 and I_2.

Example

Given: In the circuit shown above,

$$R_1 = 6\,(\Omega),\ R_2 = 10\,(\Omega),\ R_3 = 5\,(\Omega),\ V_1 = 6\ \text{(volts)},\ \text{and}\ V_2 = 12\ \text{(volts)}$$

Find: currents I_1 and I_2 using a) substitution, and b) Cramer's rule.

Solution:

Using the given values, we have two simultaneous equations: $\boxed{\begin{aligned} 11 I_1 + 5 I_2 &= 6 \\ 5 I_1 + 15 I_2 &= 12 \end{aligned}}$.

a) Substitution:

Solving the first equation for I_2 in terms of I_1, gives $\boxed{I_2 = \dfrac{6 - 11 I_1}{5}}$. Substituting this result into the second equation for I_2 gives a single equation that we can solve for I_1.

$$12 = 5 I_1 + 15 \left(\frac{6 - 11 I_1}{5} \right) = 5 I_1 + 3 \left(6 - 11 I_1 \right) = -28 I_1 + 18$$

$$\Rightarrow \boxed{I_1 = \frac{18 - 12}{28} \approx 0.2143\ \text{(amps)}} \ \text{and} \ \boxed{I_2 = \left(\frac{6 - 11 I_1}{5} \right) \approx 0.7286\ \text{(amps)}}$$

b) <u>Cramer's Rule:</u>

To use Cramer's rule, we first write the equations as a ***matrix equation***.

$$\begin{bmatrix} 11 & 5 \\ 5 & 15 \end{bmatrix} \begin{Bmatrix} I_1 \\ I_2 \end{Bmatrix} = \begin{Bmatrix} 6 \\ 12 \end{Bmatrix}$$

Then, we solve.

$$I_1 = \frac{\det \begin{bmatrix} 6 & 5 \\ 12 & 15 \end{bmatrix}}{\det \begin{bmatrix} 11 & 5 \\ 5 & 15 \end{bmatrix}} = \frac{(6 \times 15) - (5 \times 12)}{(11 \times 15) - (5 \times 5)} = \frac{30}{140} \approx 0.2143 \text{ (amps)}$$

$$I_2 = \frac{\det \begin{bmatrix} 11 & 6 \\ 5 & 12 \end{bmatrix}}{\det \begin{bmatrix} 11 & 5 \\ 5 & 15 \end{bmatrix}} = \frac{(11 \times 12) - (6 \times 5)}{(11 \times 15) - (5 \times 5)} = \frac{102}{140} \approx 0.7286 \text{ (amps)}$$