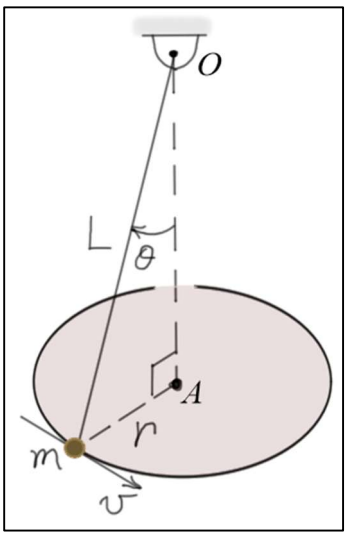
Elementary Dynamics Example #15: (Newton's Laws, Normal & Tangential Components)

Given: $mg = 10$ (lb), $T_{max} = 20$ (lb), $L = 5$ (ft)

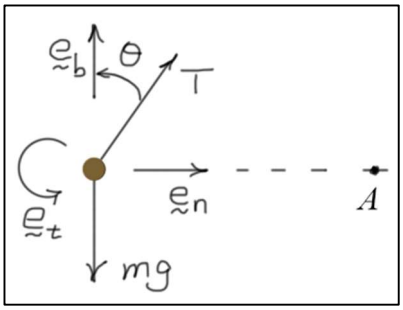
v is constant so m has ***circular motion***

Find: v_{max} the velocity of m that makes the tension $T = T_{max}$ and the corresponding angle constant angle θ



Solution: with $\boxed{T = T_{max} = 20 \text{ (lb)}}$

Free body diagram



$\boxed{\sum F_t = 0}$ (no forces in that direction)

$$\boxed{\uparrow \sum F_b = T\cos(\theta) - mg = 0}$$

$$\boxed{\rightarrow \sum F_n = T\sin(\theta) = m\left(\frac{v^2}{r}\right)}$$

Setting $T = T_{max} = 20$ (lb), we find $\boxed{\cos(\theta) = \frac{mg}{T_{max}} = \frac{10}{20}}$ $\Rightarrow$ $\boxed{\theta = \cos^{-1}\left(\frac{1}{2}\right) = 60 \text{ (deg)}}$

Using this value of θ gives

$$\boxed{m\left(\frac{v_{max}^2}{r}\right) = m\left(\frac{v_{max}^2}{L\sin(\theta)}\right) = T\sin(\theta)} \Rightarrow \boxed{v_{max} = \left(\sqrt{\frac{TL}{m}}\right)\sin(\theta) \approx 15.5403 \approx 15.5 \text{ (ft/s)}}$$