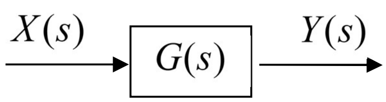
Introductory Control Systems
Block Diagrams and Transfer Functions

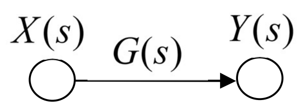
- The concepts of block diagrams and transfer functions can be used to develop ***graphical representations*** of how systems function. For example, suppose a system with transfer function $G(s)$ has input $X(s)$ and output $Y(s)$, then

$$\boxed{\frac{Y(s)}{X(s)} = G(s)} \qquad \text{or} \qquad \boxed{Y(s) = G(s)X(s)}$$

- This algebraic equation may be ***represented graphically*** as either a block diagram or a signal flow graph. In a block diagram representation, the signals are represented as "arrows" (a line segment with direction of signal flow indicated) and the process (or system) is represented by a "box" showing the transfer function. In a signal flow graph representation, the signals are represented by "nodes" and the process is indicted by an "arrow" with the transfer function indicated.



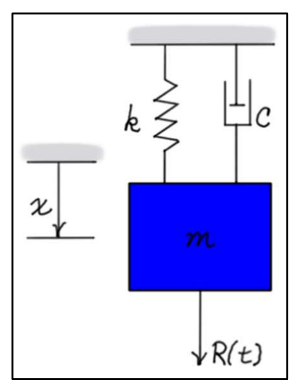
"Block Diagram"



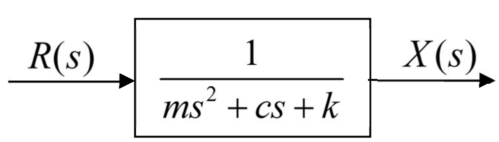
"Signal Flow Graph"

- Block diagrams and signal flow graphs can be used to represent much more complicated systems. The notes that follow focus on the use of block diagrams.
- Example: Mass-Spring-Damper

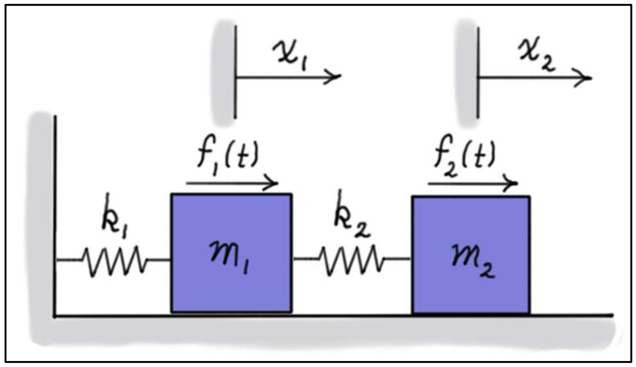
The differential equation of motion of the spring-mass-damper system has been found in previous notes to be $\boxed{m\ddot{x} + b\dot{x} + kx = f(t)}$. Applying Laplace transforms and solving for the ratio of output over input gives the transfer function $G(s)$. This, in turn, can be put into a simple block diagram as follows.



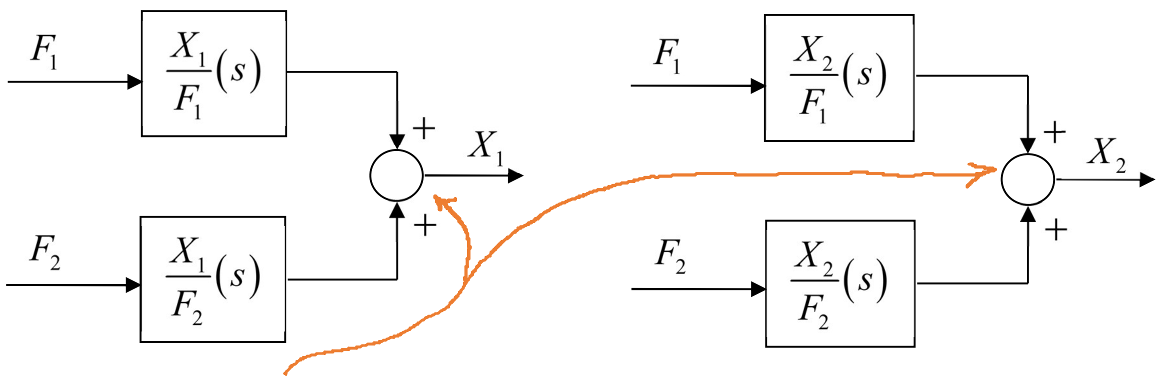
$$\frac{X(s)}{R(s)} = \frac{1}{ms^2 + cs + k} = G(s)$$



- More complex systems may have ***multiple inputs and multiple outputs***. For example, consider the two degree-of-freedom, mass-spring-damper system shown below.



- This system has two ***input forces*** $f_1(t)$ and $f_2(t)$, and it has two ***output displacements*** $x_1(t)$ and $x_2(t)$. This system can be ***represented graphically*** as follows.



The ***summation blocks*** are used to ***add*** the ***responses*** of the masses to ***each*** of the forces. Recall that the transfer functions associated with force $f_1(t)$ assume that $f_2(t)$ is zero and the transfer functions associated with force $f_2(t)$ assume that $f_1(t)$ is zero. Hence, each of the four transfer functions provide only ***part*** of the total response. The total response is the ***sum*** of the ***responses*** due to each of the input forces.

- The block diagrams above represent the algebraic equations

$$\boxed{X_1(s) = \left(\frac{X_1}{F_1}\right)F_1 + \left(\frac{X_1}{F_2}\right)F_2} \quad \text{and} \quad \boxed{X_2(s) = \left(\frac{X_2}{F_1}\right)F_1 + \left(\frac{X_2}{F_2}\right)F_2}$$

- In general, the block diagram representation of a system will contain several simple blocks connected in a ***network***. Complex networks can be reduced to simpler networks using ***block diagram reduction transformations*** discussed in the notes that follow.