

Elementary Engineering Mathematics

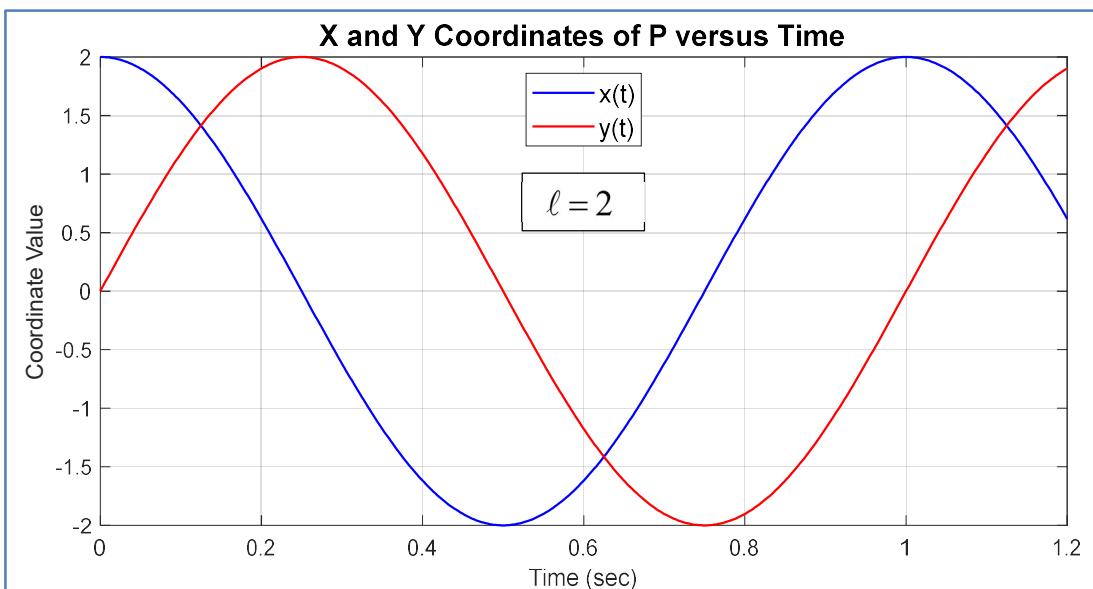
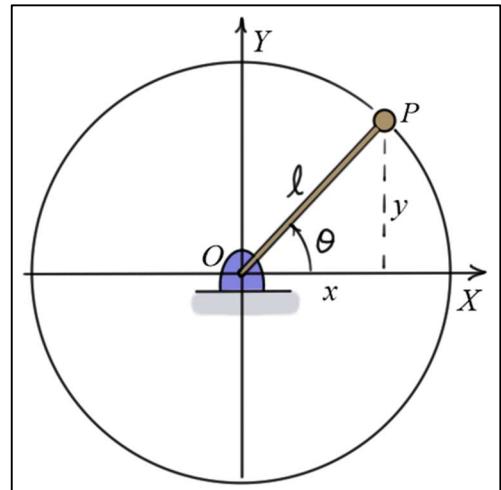
Sine and Cosine Functions of Time

Arm OP rotates so P moves in a circular path. From trigonometry, the coordinates of P are

$$x = \ell \cos(\theta) \quad y = \ell \sin(\theta)$$

If the bar completes one revolution (2π radians) in one second, then $\theta = 2\pi t$. So, we can also express the coordinates as functions of time.

$$x = \ell \cos(2\pi t) \quad y = \ell \sin(2\pi t)$$



Characteristic	Symbol
<i>Amplitude</i>	$A = \ell$
<i>Frequency</i>	$\omega = 2\pi$ (radians/sec)
	$f = \omega/2\pi$ (cycles/sec) -or- (Hertz (Hz))
<i>Period</i>	$T = 1/f$ (seconds/cycle)

Note that the sine and cosine functions (and hence the X and Y coordinates) can be related by a **phase shift**. The phase shift corresponds to a **shift in time**.

$$\sin(2\pi t) = \cos(2\pi t - \pi/2)$$

$$\cos(2\pi t) = \sin(2\pi t + \pi/2)$$

In this case, the phase shift of $\pi/2$ radians equates to a shift in time of $\frac{1}{4}$ second. For any given phase shift, the corresponding time shift depends on the frequency ω .