

Elementary Dynamics Example #16: (Newton's Laws, Normal & Tangential Components)

Given: $W_{\text{box}} = W = 5 \text{ (lb)}$, $R = 2 \text{ (ft)}$, $v_A = 4 \text{ (ft/s)}$

surface is smooth (no friction)

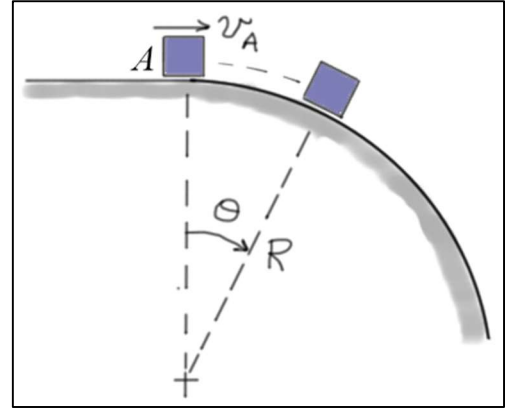
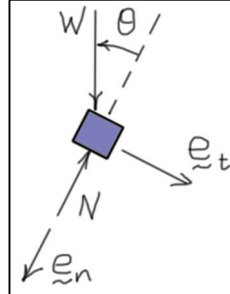
Find: $\hat{\theta}$ the angle where the box begins to leave the surface

Solution:

$$\swarrow \sum F_n = W \cos(\theta) - N = \left(\frac{W}{g}\right)\left(\frac{v^2}{R}\right)$$

$$\searrow \sum F_t = W \sin(\theta) = \left(\frac{W}{g}\right)\dot{v}$$

Free body diagram



Using the second equation,

$$\frac{dv}{dt} = \frac{dv}{d\theta} \frac{d\theta}{dt} = \frac{v}{R} \frac{dv}{d\theta} = g \sin(\theta) \Rightarrow \int v dv = \int g R \sin(\theta) d\theta \Rightarrow \frac{1}{2} v^2 = -gR \cos(\theta) + D$$

using indefinite integrals

Initial condition: @ $\theta = 0$, $v = v_A = 4 \text{ (ft/s)}$ $\Rightarrow D = \frac{1}{2} 4^2 + gR \cos(0) = 8 + 2g$

$$\Rightarrow v^2 = 2(8 + 2g - 2g \cos(\theta)) \quad (R = 2 \text{ (ft)})$$

When the box begins to leave the surface, $N = 0$:

$$\sum F_n = W \cos(\hat{\theta}) - N = W \cos(\hat{\theta}) = \left(\frac{W}{g}\right)\left(\frac{v^2}{R}\right) = \left(\frac{W}{g}\right)\left(\frac{v^2}{2}\right) = \left(\frac{W}{g}\right)(8 + 2g - 2g \cos(\hat{\theta}))$$

$$\Rightarrow (3W) \cos(\hat{\theta}) = \left(\frac{W}{g}\right)(8 + 2g) \Rightarrow \hat{\theta} = \cos^{-1}\left(\frac{8 + 2g}{3g}\right) = 41.454 \approx 41.5 \text{ (deg)}$$